Expressions Review: Applications and Synthesis

MEDIUM

1. Cube Radicals

Multiply and simplify completely.

$$\sqrt[3]{12x^2} \cdot \sqrt[3]{18x^4}$$

2. Binomial Structures

Simplify completely. Choose an efficient multiplication strategy.

$$(2+\sqrt{3})^2 - (2-\sqrt{3})(2+\sqrt{3})$$

3. General Form to Vertex Form

Consider $y = -2x^2 + 8x - 3$.

a) Complete the square to express the function in vertex form.
b) Identify the vertex.
c) State the range.

4. Division and Original Restrictions

Consider $\frac{x^2-9}{x^2+5x+6} \div \frac{x-3}{x+2}$.

a) State all NPVs from the original expression, including values that make the divisor zero.
b) Simplify completely.

5. Reconstructing a Terminal Arm MEDIUM

An angle θ satisfies $\tan\theta = -\frac{3}{4}$ and $\sin\theta > 0$.

a) Determine the quadrant.
b) Give one possible point with integer coordinates on its terminal arm.
c) Determine the exact value of $\boldsymbol{r}$.
d) Determine the exact value of $\boldsymbol{\cos\theta}$.
e) Give a different integer-coordinate point on the same terminal arm. Explain why the three trigonometric ratios remain unchanged.

SPICY

6. Strategic Rationalization

Rationalize the denominator and simplify completely.

$$\frac{\sqrt{12}+2}{4-2\sqrt{3}}$$

7. Factored Form to Vertex Form

Consider $y = (x - 2)(x + 3)$.

a) Expand the function into general form.

b) Complete the square to express it in exact vertex form.

c) Explain why the axis of symmetry agrees with the midpoint of the zeros.

8. Reconstructing and Comparing Quadratics

A quadratic function has vertex $(1, -8)$ and passes through $(0, -6)$.

a) Determine $\boldsymbol{f(x)}$ in vertex form.

b) Rewrite $\boldsymbol{f(x)}$ in general form.

c) Factor to determine the zeros.

d) A second quadratic is defined by $\boldsymbol{g(x) = -f(x)}$. Describe the geometric relationship between the graphs.

9. Factoring by Structure

Factor each expression completely.

a) $\boldsymbol{2(x-1)^2 - 5(x-1) - 3}$

b) $\boldsymbol{x^4 - 10x^2 + 9}$

10. Unlike Denominators

Consider $\frac{3}{x^2-4} - \frac{1}{(x+2)^2}$.

a) State all NPVs.

b) Determine the least common denominator (LCD).

c) Combine and simplify completely.

11. Mixed Rational Operations

Consider $\left(\frac{x^2-4}{x^2+3x+2} \cdot \frac{x+3}{x-2}\right) - \frac{1}{x+1}$.

a) State all NPVs from the original expression.

b) Simplify completely, choosing an efficient order of operations.

12. Symbolic Reflections

Suppose $\sin 38^\circ = s$, $\cos 38^\circ = c$, and $\tan 38^\circ = t$, where $s, c, t > 0$.

a) Express $\boldsymbol{\sin 142^\circ}$ in terms of $\boldsymbol{s}$.

b) Express $\boldsymbol{\tan 218^\circ}$ in terms of $\boldsymbol{t}$.

c) Express the common value of $\boldsymbol{\cos 142^\circ}$ and $\boldsymbol{\cos 218^\circ}$ in terms of $\boldsymbol{c}$.

d) Explain geometrically why these two cosine values are equal.

EXTRA-SPICY

13. Complex Rational Expressions

Consider the complex rational expression below.

$$\left(\frac{x+1}{x-2}-\frac{x-1}{x+2}\right)\div\left(\frac{x^2-1}{x^2-4}\div\frac{x+1}{x-2}\right)$$

a) Determine all NPVs, including values that make the entire denominator equal to zero.

b) Simplify completely, preserving all original restrictions.

14. Exact-Value Synthesis

Angles α and β lie between 0° and 360°.

Angle α lies in Quadrant III and satisfies $\cos\alpha = -\frac{\sqrt{3}}{2}$.

Angle β lies in Quadrant IV and satisfies $\tan\beta = -1$.

a) Determine the exact measure of $\boldsymbol{\alpha}$.

b) Determine the exact measure of $\boldsymbol{\beta}$.

c) Evaluate exactly, without decimals: $\frac{\sin\alpha-\tan\beta}{\cos\alpha}$

d) Find a different standard-position angle with the same sine as $\boldsymbol{\alpha}$ but a different cosine. Justify geometrically.