Expressions Review: Foundations and Connections

Instructions: Show all work. Simplify completely and use exact values.

MILD

1. Exponent Laws

Assume $x > 0$ and $y > 0$.

a) Rewrite using a rational exponent: $\frac{1}{\sqrt[3]{x^2}}$

b) Simplify completely using positive exponents: $\frac{(2x^{-2}y^3)^2}{4x^{-1}y^2}$

2. Evaluating Rational Exponents

Evaluate each expression exactly without a calculator.

a) $\mathbf{216^{\frac{2}{3}}}$ b) $(-256)^{\frac{2}{3}}$ c) $\left(\frac{1}{25}\right)^{-\frac{1}{2}}$

3. Standard Position and Reference Angles

a) Sketch $250°$ in standard position, including its counterclockwise rotation arc.

b) Determine its reference angle.

c) An angle has a reference angle of $35°$ and lies in Quadrant IV. Determine its standard-position measure between $0°$ and $360°$.

MEDIUM

4. Radical Operations

Simplify completely.

a) $\sqrt{18}\sqrt{8} - \sqrt{75} + \sqrt{12}$

b) Rationalize the denominator and simplify: $\frac{2}{3-\sqrt{5}}$

5. Vertex Form

Consider $y = -(x - 2)^2 + 9$

a) Identify the vertex.

b) Calculate the y-intercept.

c) Sketch the parabola on a coordinate system you construct. Label the vertex, y-intercept, and its reflection across the axis of symmetry.

d) State the range.

6. Factored and General Form

Consider $y = 2(x + 4)(x - 2)$

a) Determine the zeros.

b) Calculate the axis of symmetry.

c) Determine the vertex.

d) Rewrite the function in general form.

7. Recognizing Factoring Structures

Factor each expression completely.

a) $6x^2 - 24$ b) $3x^2 + 8x + 4$

8. Simplifying Rational Expressions

Consider $\frac{x^2 - 16}{x^2 + x - 12}$

a) Determine all non-permissible values (NPVs).
b) Factor and simplify completely.

9. Combining Rational Expressions

Consider $\frac{2}{x+1} - \frac{1}{x^2-1}$

a) State all non-permissible values (NPVs).
b) Determine the least common denominator (LCD).
c) Combine and simplify completely.

10. Coordinate-Based Trigonometric Ratios

A terminal arm passes through $P(-5, 12)$.

a) Determine the exact distance r from the origin to the point.
b) Determine $\sin\theta$.
c) Determine $\cos\theta$.
d) Determine $\tan\theta$.
e) Reflect P across the y-axis. Which trigonometric ratio remains unchanged? Explain using coordinates.

SPICY

11. Completing the Square

Consider $y = 2x^2 - 6x + 1$

a) Complete the square to rewrite the function in vertex form. Keep all values exact.
b) Identify the exact vertex.

12. Exact Values and Reflections

Determine each exact value, then explain the final relationship.

a) $\sin 240°$ b) $\cos 315°$ c) $\tan 330°$

d) Explain geometrically why $\tan 210° = \tan 30°$ even though the angles lie in different quadrants.