Special Angle Problems

MILD — Using the exact values we constructed

1. Use the special-triangle geometry from your investigations to determine each exact value.
   a) $\cos 45°$
   b) $\tan 30°$
   c) $\sin 60°$

2. A learner examines the $60°$ bowtie and claims that $\tan 60° = \frac{1}{\sqrt{3}}$ because the horizontal component is 1 and the vertical component is $\sqrt{3}$.
   a) Identify the learner's mistake.
   b) Determine the correct exact value of $\tan 60°$.
   c) Explain how the $\tan 60°$ and $\tan 30°$ right triangles are related.

MEDIUM — Carrying exact values into other quadrants

3. Consider $225°$.
   a) Sketch $225°$ in standard position.
   b) Identify its reference angle.
   c) Determine $\sin 225°$, $\cos 225°$, and $\tan 225°$ exactly.
   d) Identify a *different* angle in $0° \leq \theta < 360°$ with $\cos\theta = -\frac{1}{\sqrt{2}}$. Explain why its cosine equals $\cos 225°$.

4. Consider $300°$.
   a) Determine $\cos 300°$ and $\tan 300°$.
   b) Identify its reference angle.
   c) Identify a different angle in $0° \leq \theta < 360°$ with the same sine value as $300°$. Explain why they have the same sine value.

5. Consider $150°$.
   a) Identify its reference angle.
   b) Determine $\sin 150°$ and $\cos 150°$.
   c) Explain why one of these trigonometric values remains unchanged by the reflection while the other becomes its opposite.

SPICY — Mixed exact values and reverse reasoning

6. Determine each exact value, choosing the reference angle and appropriate sign.
   a) $\sin 315°$
   b) $\cos 240°$
   c) $\tan 210°$
   d) $\tan 135°$

7. Consider terminal arms on the coordinate axes.
   a) Determine $\cos 180°$ and $\sin 270°$ exactly using x, y, and r.
   b) Explain why $\tan 90°$ and $\tan 270°$ are undefined using $\tan\theta = \frac{y}{x}$.

8. For each part, identify **the possible** standard-position angles in $0° \leq \theta < 360°$. Sketch each angle in standard position
   a) $\tan\theta = -1$
   b) $\sin\theta = \frac{1}{\sqrt{2}}$
   c) $\tan\theta = \sqrt{3}$

9. An angle θ satisfies $\sin\theta = -\frac{\sqrt{3}}{2}$ and $\cos\theta = -\frac{1}{2}$, where $0° \leq \theta < 360°$.
   a) Determine its quadrant. Explain using x, y, and r.
   b) Determine its reference angle and the exact measure of θ.
   c) Determine $\tan\theta$ exactly.
   d) Explain why both given values together are needed determine a unique angle in the interval, whereas either value alone does not.

EXTRA SPICY — Combining, comparing, and justifying exact values

10. Evaluate the expression exactly. Determine each reference angle and sign independently, then simplify without decimals.

$$\sin 135° + \cos 240° - \tan 120°$$

11. Compare each pair without using decimals. Justify your conclusions using exact values.
   a) Which is greater, $\sin 120°$ or $\cos 315°$?
   b) Which is greater, $\tan 150°$ or $\cos 135°$?

12. Two learners are examining $150°$ and $210°$.

   Learner A: "Both angles have a reference angle of $30°$, so their sine values must be equal."
   Learner B: "Their cosine values are equal because the terminal arms are related by reflection across the x-axis."

   a) Determine the exact sine and cosine values of both angles.
   b) Evaluate each learner's claim.
   c) Explain geometrically why one trigonometric value is preserved by this reflection while the other changes sign.