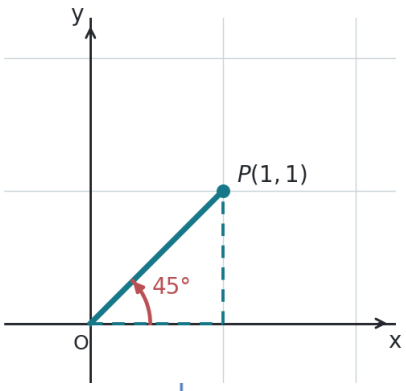
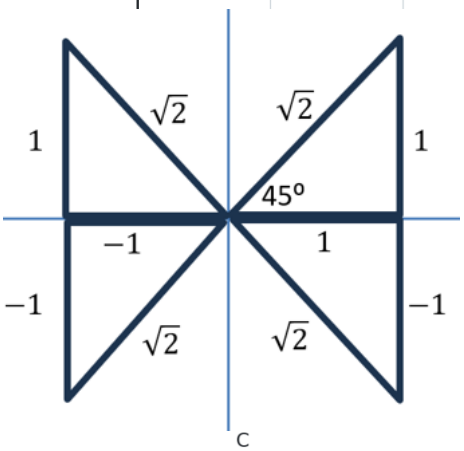
# Special Angles Investigation

## 1. Investigating $45^\circ$




The diagram shows a point $P(1,1)$ on the terminal arm of $45^\circ$.

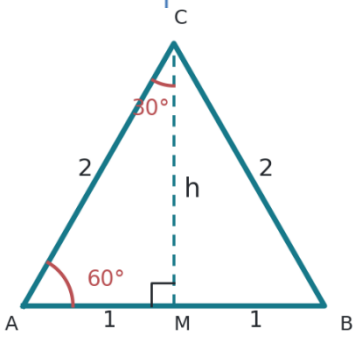
a) Identify the horizontal and vertical components. Explain why the associated triangle has angles of $45^\circ$, $45^\circ$, and $90^\circ$.
b) Use the Pythagorean theorem to determine the exact distance $r$ from the origin to $P$ in radical form.
c) Determine the exact value of $\sin 45^\circ$, $\cos 45^\circ$, and $\tan 45^\circ$.
d) Explain geometrically why $\sin 45^\circ = \cos 45^\circ$.




**Bowtie investigation** — $45^\circ$ in four quadrants

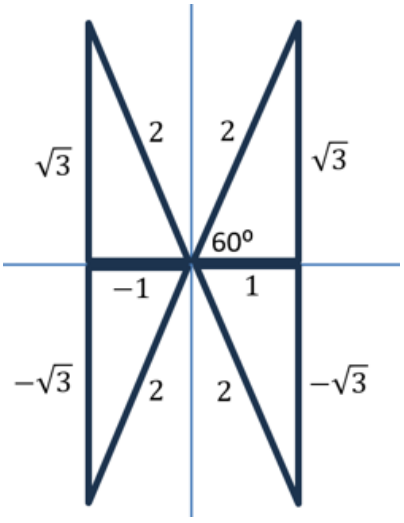
e) Label the other three standard-position angles represented by the bowtie. Include counterclockwise rotation arcs.
f) Determine $\sin 135^\circ$ and $\tan 315^\circ$. Explain how these values are related to $\sin 45^\circ$ and $\tan 45^\circ$.

## 2. Investigating $30^\circ$ and $60^\circ$




The equilateral triangle has three sides of length 2. Its altitude meets the base at $M$.

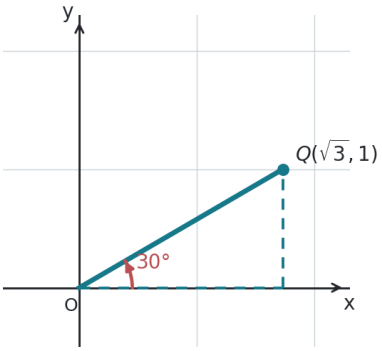
a) Explain how an equilateral triangle relates to a $60^\circ$ and a $30^\circ$ angle.
b) Use the Pythagorean theorem to determine the exact altitude $h$ in radical form.
c) Using one of the right triangles, determine the exact values of $\sin 60^\circ$, $\cos 60^\circ$, and $\tan 60^\circ$.
d) Determine the exact values of $\sin 30^\circ$, $\cos 30^\circ$, and $\tan 30^\circ$,

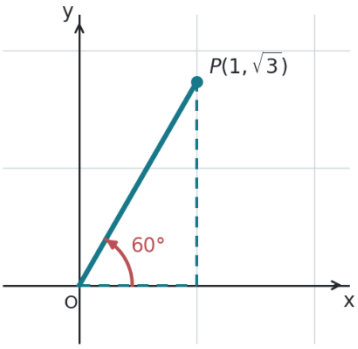



**Bowtie investigation** — $60^\circ$ in four quadrants

e) Label the other three standard-position angles represented by the bowtie. Include their counterclockwise rotation arcs.
f) Compare $\sin 60^\circ$ with $\sin 120^\circ$, then $\cos 60^\circ$ with $\cos 120^\circ$. Which values are equal and which are opposites? Explain using signed coordinates.

**3. Connecting the triangles to coordinates**




Consider the points $P(1, \sqrt{3})$ and $Q(\sqrt{3}, 1)$.

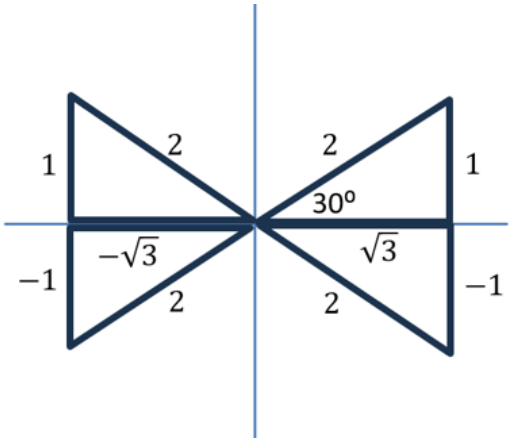
a) Identify which point represents $30^\circ$ and which represents $60^\circ$.

b) Using the Pythagorean theorem, determine the length of each hypotenuse.

c) Using the Pythagorean theorem, show that the two points are the same distance from the origin.

d) Explain how exchanging the horizontal and vertical components changes sine and cosine. Why does $\sin 30^\circ = \cos 60^\circ$ and $\cos 30^\circ = \sin 60^\circ$?

**Bowtie investigation** — $30^\circ$ in four quadrants



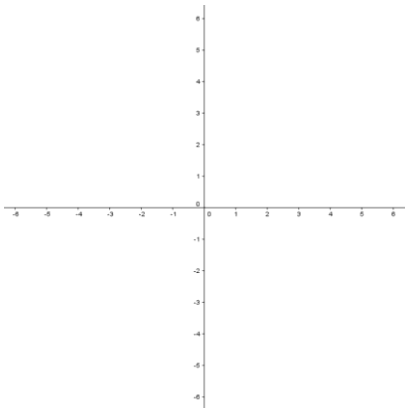

a) Label the four standard-position angles represented by the bowtie. Add counterclockwise rotation arcs for each terminal arm.

b) Determine $\cos 210^\circ$ and $\sin 330^\circ$ exactly. Explain how the relevant coordinate determines the sign of each value.

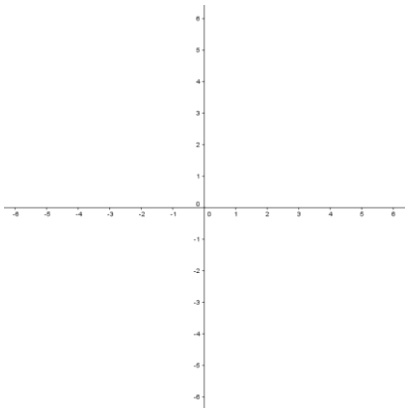
c) Two terminal arms in this bowtie produce the same negative tangent value. Identify both angles and explain their geometric relationship.

**Trigonometric Values:**

Indicate whether the value of **sin $\theta$** will be negative or positive in each quadrant.



Indicate whether the value of **cos $\theta$** will be negative or positive in each quadrant.



Indicate whether the value of **tan $\theta$** will be negative or positive in each quadrant.

