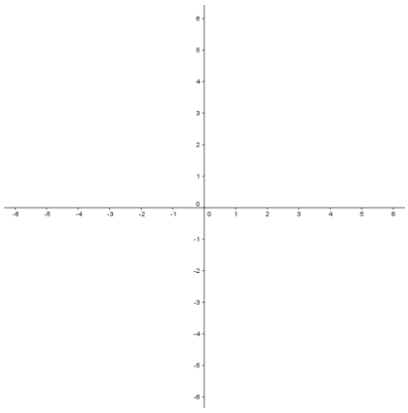
Trigonometric Patterns

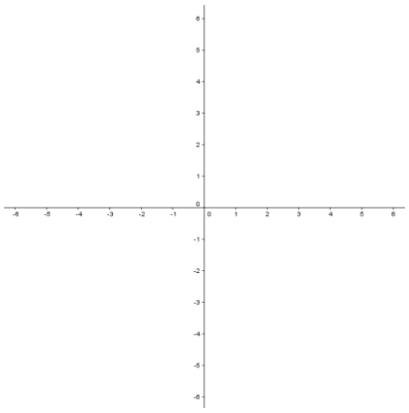
Use geometric reflections and coordinate changes to predict related trigonometric values.

Trigonometric Values GeoGebra Investigation: https://www.geogebra.org/m/adpfsy3g

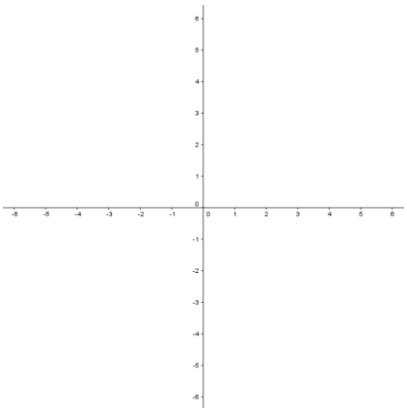
Indicate whether the value of **sin θ** will be negative or positive in each quadrant.



Indicate whether the value of **cos θ** will be negative or positive in each quadrant.



Indicate whether the value of **tan θ** will be negative or positive in each quadrant.



Consider the angle pair $37°$ and $143°$.

a. Identify the reflection connecting $37°$ and $143°$. Explain why it produces predictable trig relationships using points the same distance from the origin.
b. A learner claims that **any** Quadrant I and Quadrant II angles must have **equal sine values** and **opposite cosine and tangent values**. Is the learner correct? Test this with $37°$ and $128°$. What would a correct statement say?

MILD — Reading coordinate changes
Coordinates GeoGebra Investigation: https://www.geogebra.org/m/uwz74vej

1. Compare $P(3, 4)$ with $Q(-3, 4)$.
   a. Identify the reflection relating these two coordinates.
   b. Determine the sine, cosine and tangent values for both P and Q.
   c. Identify which trigonometric values are equal and which are opposites.

2. Compare Point $R(3, -4)$ with $P(3, 4)$.
   a) Identify the reflection and the coordinate that stays unchanged.
   b) Determine the three trigonometric values for R and P.
   c) Explain why cosine is unchanged compared to P using the values of x, y, and r.

3. Compare Point $S(-3, -4)$ with $P(3, 4)$.
   a. Identify the transformation connecting the points.
   b. Which trig value stays unchanged, verify using the x, y, and r.
   c. Explain why it stays unchanged despite the two negative coordinates.

MEDIUM — Working without diagrams

4. An angle measures $58°$ in standard position.
   a. Determine its reflected angle across the y-axis.
   b. Determine its reflected angle across the x-axis.
   c. Determine the angle obtained through the origin.

5. Igor knows $\cos 36° \approx 0.809$. He reflects the terminal arm across the y-axis to obtain $144°$ and claims $\cos 144° \approx 0.809$.
   a. Use the angle relationship to explain Igor's error.
   b. Determine the correct value of $\cos 144°$.
   c. Find the angle produced by reflection across the x-axis and explain why its cosine equals $\cos 36°$.

SPICY — Evaluating related values

6. Draw $28°$. If $\sin 28° \approx 0.4695$:
   a. Draw $152°$. Determine the value of $\sin 152°$ without recalculating.
   b. Draw $332°$. Determine the value of $\sin 332°$ without recalculating.

7. Draw $41°$. If $\cos 41° = \text{k}$ and $\tan 41° = \text{m}$:
   a. Draw $319°$. Determine the value of $\cos 319°$.
   b. Determine the value of $\tan 139°$.
   c. Draw $221°$. Determine the value of $\cos 221°$.
   d. Determine the value of $\tan 221°$.

8. Draw $137°$. If $\sin 137° = \text{a}$ and $\cos 137° = -\text{b}$, determine each expression in terms of a and b:
   a. $\sin 223°$. Explain the geometric relationship to $\sin 137°$.
   b. $\cos 43°$. Explain the geometric relationship to $\cos 137°$.
   c. If $\tan\theta = \frac{\sin\theta}{\cos\theta}$, what is the value of $\tan 137°$?
   d. Draw $317°$. Determine the value of $\tan 317°$.

EXTRA SPICY — Explaining and combining patterns

9. A learner starts with $\text{P}(-4, 3)$, reflects it across the y-axis, and then reflects the resulting point across the x-axis. The learner claims that performing two reflections returns all three trigonometric ratios to their original values.
   a. Determine the coordinates after each reflection.
   b. Identify the single transformation with the same overall effect.
   c. Determine exact sine, cosine and tangent values for the original and final terminal arms.
   d. Evaluate the learner's claim. Which values remain unchanged, and which become their opposites? Explain.

10. Let $P(x, y)$ lie in Quadrant I, where $x > 0, y > 0$ and $r = \sqrt{x^2 + y^2}$.
    a. Give the coordinates obtained by reflecting P across the y-axis, prove which trig value stays unchanged and determine what happens to the other two.
    b. Give the coordinates obtained by reflecting P across the x-axis, prove which trig value stays unchanged and determine what happens to the other two.
    c. Give the coordinates obtained by reflecting P through the origin, prove which trig value stays unchanged and determine what happens to the other two.
    d. Prove that all three transformations preserve the distance r using the Pythagorean theorem.

11. Four standard-position angles share a reference angle of 29°. Let $\sin 29° = s$, $\cos 29° = c$ and $\tan 29° = t$, where $s > 0, c > 0$ and $t > 0$.
    a. Determine all four standard-position angles in the interval $0° \leq \theta < 360°$.
    b. For each angle, express sine, cosine and tangent in terms of s, c and t, including appropriate signs.