Angle Reflections

Use geometric reflections and coordinate changes to predict related trigonometric values.

GeoGebra Investigation

Use the exact GeoGebra app: https://www.geogebra.org/m/adpfsy3g

A. GeoGebra — Finding Trigonometric Values

1. Use GeoGebra to determine an approximate value for each expression. Record values to two decimal places.

   a. $\tan 60° =$ _____
   b. $\sin 230° =$ _____
   c. $\cos 110° =$ _____
   d. $\tan 250° =$ _____
   e. $\sin 170° =$ _____
   f. $\cos 340° =$ _____

B. GeoGebra — How the Values Change as the Angle Increases

2. The Pattern of SINE. Use GeoGebra to watch the sine value as the angle changes within each interval. State whether the value increases or decreases. State the range of values for each.

   a. As θ increases from 0° to 90°, what happens to sin θ? _____
   b. As θ increases from 90° to 180°, what happens to sin θ? _____
   c. As θ increases from 180° to 270°, what happens to sin θ? _____
   d. As θ increases from 270° to 360°, what happens to sin θ? _____

3. The Pattern of COSINE. Repeat the observation for cosine.

   a. As θ increases from 0° to 90°, what happens to cos θ? _____
   b. As θ increases from 90° to 180°, what happens to cos θ? _____
   c. As θ increases from 180° to 270°, what happens to cos θ? _____
   d. As θ increases from 270° to 360°, what happens to cos θ? _____

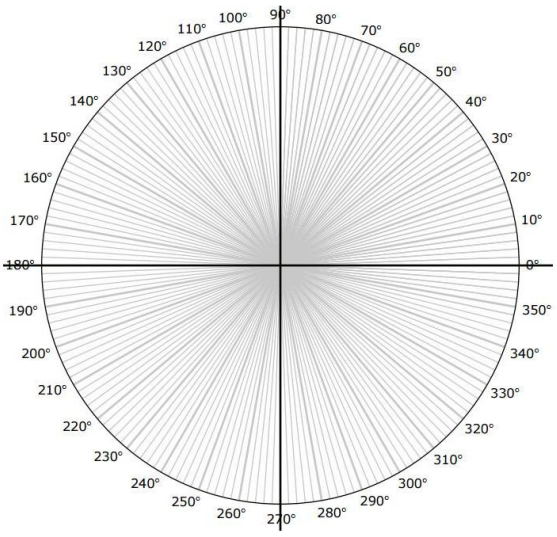
4. The Pattern of TANGENT. Watch the tangent value carefully, particularly near 90° and 270°, where it is undefined.

   a. As θ increases from 0° toward 90°, what happens to tan θ? _____
   b. As θ increases from just above 90° toward 180°, what happens to tan θ? _____
   c. As θ increases from 180° toward 270°, what happens to tan θ? _____
   d. As θ increases from just above 270° toward 360°, what happens to tan θ? _____
   e. Investigate tangent near 90° and 270° from both sides. Does it have a largest or smallest value where defined? Explain.

For each given value, use the app to locate two terminal arms between 0° and 360° that give approximately that value. Record their angles to the nearest degree. **Color each matching pair of arms with one color, using a different color for each pair.** Identify geometric patterns from your completed circle.

C — Sine: What makes sine stay the same?

5. Find and color the two matching terminal arms for each value.
   a) $\sin\theta \approx 0.46$ Angles: _____ and _____
   b) $\sin\theta \approx 0.96$ Angles: _____ and _____
   c) $\sin\theta \approx -0.92$ Angles: _____ and _____
   d) $\sin\theta \approx -0.43$ Angles: _____ and _____

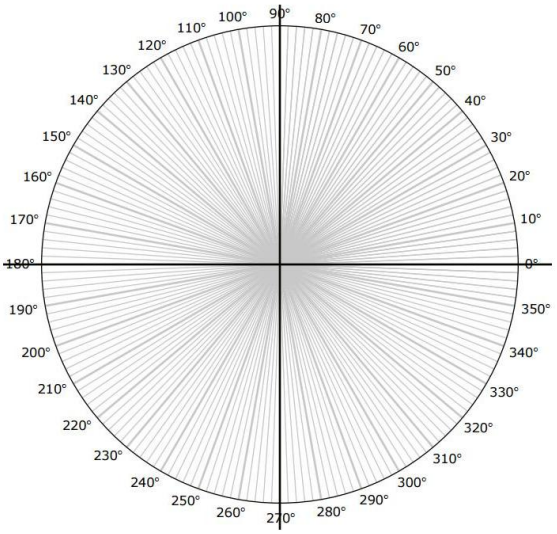


6. What reflection connects the two terminal arms in each colored pair?

7. Predict what happens to cosine and tangent under this reflection. Explain why sine remains unchanged.

D — Cosine: What makes cosine stay the same?

8. Find and color the two matching terminal arms for each value.
   a) $\cos\theta \approx 0.30$ Angles: _____ and _____
   b) $\cos\theta \approx 0.91$ Angles: _____ and _____
   c) $\cos\theta \approx -0.66$ Angles: _____ and _____
   d) $\cos\theta \approx -0.99$ Angles: _____ and _____

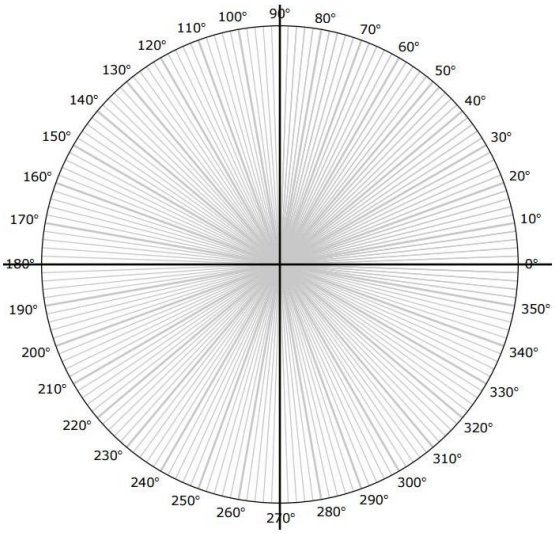


9. What reflection connects the two terminal arms in each colored pair?

10. Explain why cosine remains unchanged while sine and tangent change sign.

E — Tangent: Why is this pattern different?

11. Find and color the two matching terminal arms for each value.
   a) $\tan\theta \approx 1.88$ Angles: _____ and _____
   b) $\tan\theta \approx 0.36$ Angles: _____ and _____
   c) $\tan\theta \approx -0.21$ Angles: _____ and _____
   d) $\tan\theta \approx -2.47$ Angles: _____ and _____



12. What reflection connects the two terminal arms in each colored pair?

13. Explain why tangent is unchanged, even though sine and cosine change sign.

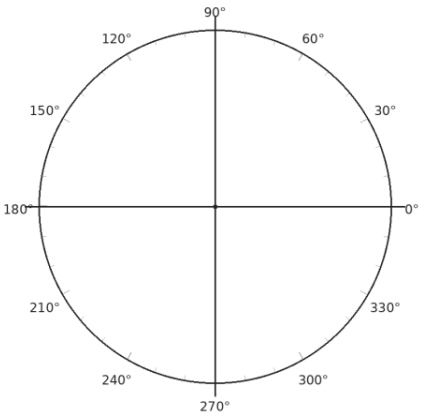
C. Check Your Coordinate-Sign Reasoning

14. For each claim, write Always True, Sometimes True, or Never True, and explain your answer using coordinates.

    a. A cosine value for an angle in Quadrant II is positive. _____
    b. A tangent value for an angle in Quadrant III is negative. _____
    c. A sine value for an angle in Quadrant III is negative. _____
    d. A cosine value for an angle in Quadrant I is positive. _____
    e. A positive tangent value means the terminal arm is in Quadrant II. _____
    f. A negative sine value means the terminal arm is in Quadrant III or IV. _____

D. Sine — Find & Draw Related Angles

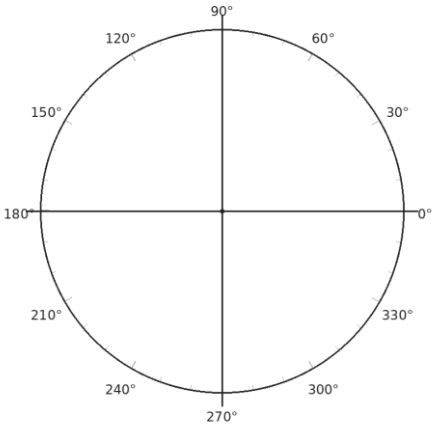
Tutorial: $\sin\theta = 0.8$



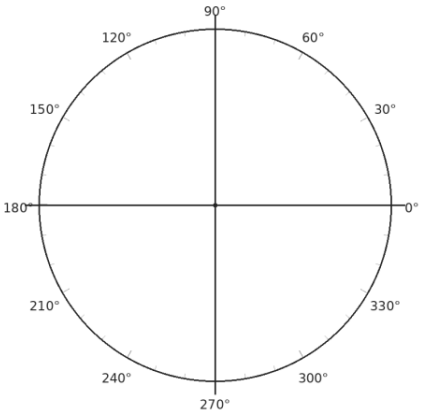
    a. Use the inverse-sine function $\sin^{-1}(x)$ on Desmos to find a first angle. _____
    b. Draw that terminal arm on your circle.
    c. Reflect it across the appropriate axis.
    d. Calculate the value of the second angle using angle relationships.

15. For each value below, use $\sin^{-1}(x)$ to find the first angle. Then, use angle relationships to find the second angle solution.

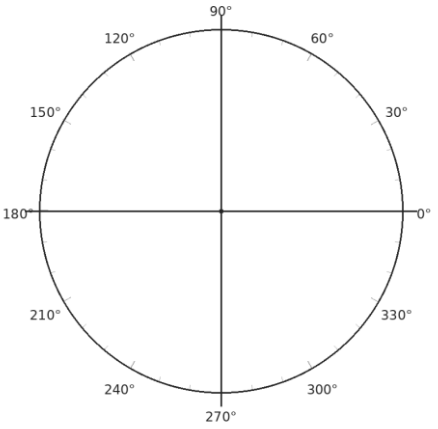
$\sin\theta = 0.5$



$\sin\theta = -0.61$



$\sin\theta = 1$

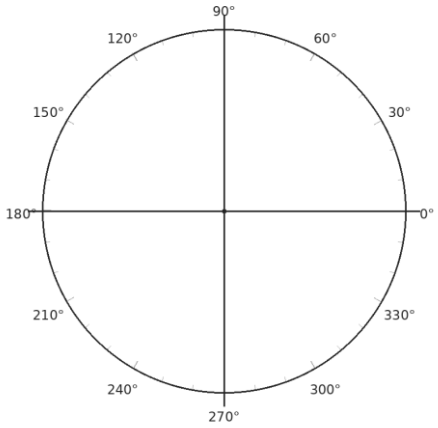


16. Explain why the final sine equation gives only one angle instead of two in the interval from 0° to 360°.

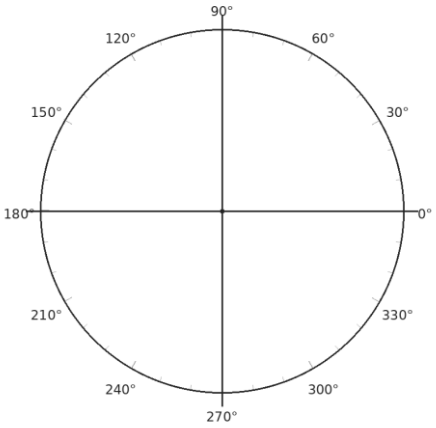
E. Cosine — Find & Draw Related Angles

17. For each value below, use $\cos^{-1}(x)$ to find the first angle. Then, use angle relationships to find the second angle solution.

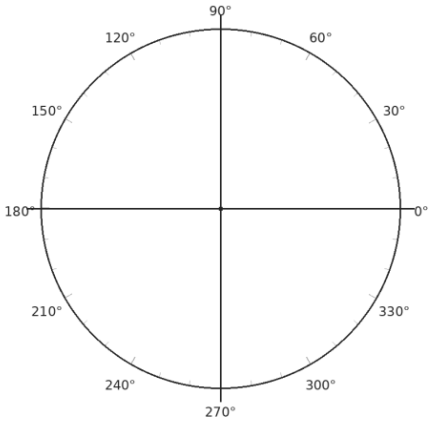
$\cos\theta = 0.3$



$\cos\theta = -0.45$



$\cos\theta = -1$

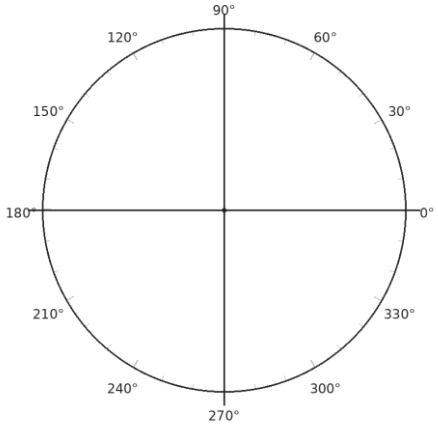


18. Explain why the final cosine equation gives only one angle instead of two.

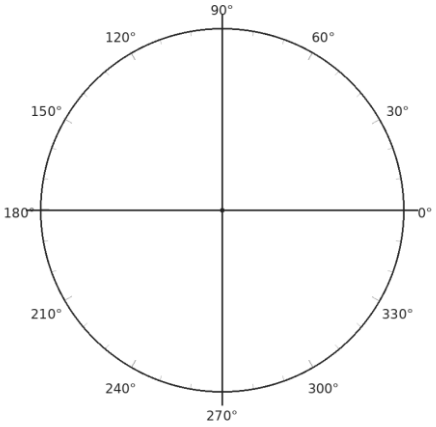
F. Tangent — Find & Draw Related Angles

19. For each value below, use $\tan^{-1}(x)$ to find the first angle. Then, use angle relationships to find the second angle solution.

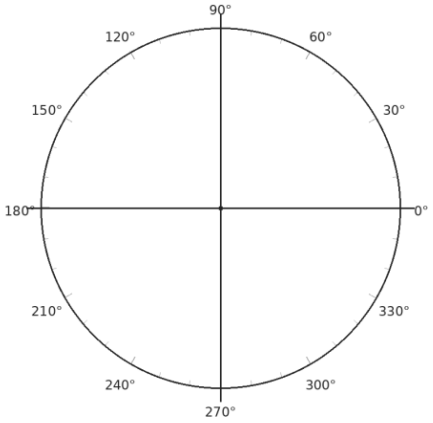
$\tan\theta = 1.6$



$\tan\theta = -0.9$



$\tan\theta = 25$



20. Before drawing, describe the transformation that preserves tangent. Is it a reflection across an axis? Explain briefly.