Reference Angles

A **reference angle** (α) is the acute angle formed between a terminal arm and the x-axis: $0° < \alpha < 90°$. For this lesson, standard-position angles lie from 0° to 360°. Quadrantal angles do not form an acute reference angle.

GeoGebra Investigation — Discovering Reference Angles: https://www.geogebra.org/m/jqgsauqf
Use the app to change the standard-position angle. Record what you notice before describing a general rule.

1. Set the angle to 40° and 140°. Compare the acute angles formed between each terminal arm and the x-axis. What remains the same, and what changes?

2. Set the angle to 120°, 150°, and 175°. Record the acute angle each time. As the terminal arm approaches the negative x-axis, what happens to the acute angle?

3. Investigate 220° and 320°. Which direction along the x-axis forms an acute angle with each terminal arm? Why are the acute angles equal?

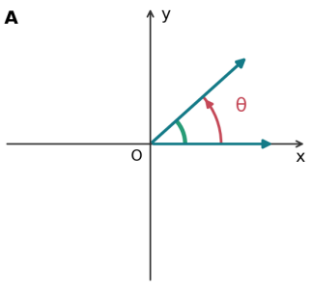
4. Investigate 0°, 90°, 180°, 270°, and 360°. Which positions do not form an acute angle with the x-axis? Explain.

5. Find one angle in each quadrant whose terminal arm makes an acute angle of 25° with the x-axis. Record all four standard-position angles.

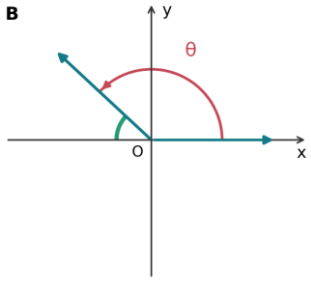
6. Using your observations, describe what a **reference angle** is. Identify the terminal arm, the relevant direction along the x-axis, and why the angle must be acute.

MILD — Recognize the Geometry

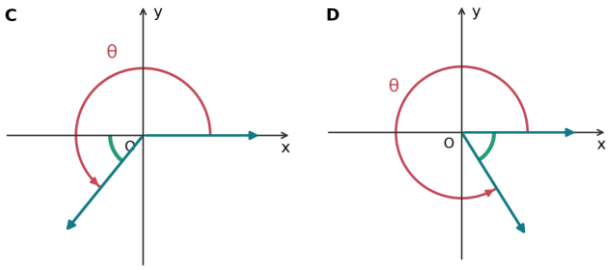
7. Diagram A shows an angle in standard position in Quadrant I. Without measuring, explain why the standard-position angle and the reference angle have equal measures.



8. Diagram B shows a standard-position angle of 137°. Determine its reference angle. Explain why the acute angle is measured to the negative x-axis.



9. Diagram C represents 231°, and Diagram D represents 302°. Determine each reference angle. For each, identify which direction along the x-axis forms the acute angle.



MEDIUM — Determine and Reconstruct

10. Without sketching first, determine the reference angle for each standard-position angle: a) 128° b) 236° c) 319°.

11. A learner claims that the reference angle for 239° is 121° because $360° - 239° = 121°$. Explain the mistake and determine the correct reference angle.

12. A reference angle measures 42°. Determine the standard-position angle if its terminal arm lies in a) Quadrant I or b) Quadrant II.

13. A reference angle measures 57°. Determine the standard-position angle if its terminal arm lies in a) Quadrant III or b) Quadrant IV. Explain which direction of the x-axis forms the acute angle in each case.

SPICY — Choose the Direction of Reasoning

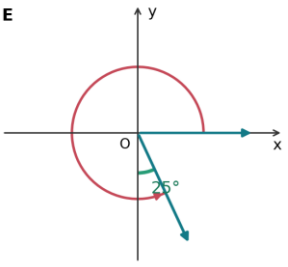
14. Find the unknown measure in each case. Decide whether to find a reference angle or reconstruct a standard-position angle:
a) $\theta = 264°$ — find α. b) $\alpha = 28°$, Quadrant IV — find θ.
c) $\theta = 103°$ — find α. d) $\alpha = 64°$, Quadrant III — find θ.

15. Consider 41°, 139°, 221°, 319°, 321°. Four angles share one reference angle. Identify the four angles, determine their shared reference angle, and explain why the remaining angle does not belong.

16. A reference angle measures 37°. Determine every standard-position angle from 0° to 360° with this reference angle. Sketch two of them in different quadrants, including both arcs. Explain why there are exactly four possibilities.

EXTRA SPICY — Reason from Incomplete Information

17. Diagram E shows a terminal arm in Quadrant IV. The marked 25° is measured from the negative y-axis, not the x-axis. Determine the standard-position angle and its reference angle. Explain why 25° is not the reference angle.



18. An unknown angle has a reference angle of 34°. Its standard-position measure satisfies $100° < \theta < 340°$, and its terminal arm is not in Quadrant III. Find **every possible value** and explain why there are no others.