Complex Rational Expressions

Select efficient methods to simplify these complex rational expressions completely and state all NPVs. The mathematics is not new; the operations are nested.

MILD — Recognize the large fraction as division

1. $\dfrac{\frac{2}{x}}{\frac{3}{x+1}}$

2. $\dfrac{\frac{x^2-4}{x+3}}{\frac{x+2}{x-1}}$

MEDIUM — Complete an operation in the upper expression

3. $\dfrac{\frac{1}{x-1}+\frac{1}{x+2}}{\frac{3}{x-1}}$

4. $\dfrac{\frac{x-1}{x-2}-\frac{1}{x+2}}{\frac{x}{x^2-4}}$

SPICY — Coordinate operations in both layers

5. $\dfrac{\frac{1}{x+4}+\frac{1}{x-4}}{\frac{x}{x^2-16}+\frac{1}{x+4}}$

6. In #5, one learner combines the upper and lower expressions separately. Another multiplies both by $x^2 - 16$. Why are both methods valid? Which is more efficient here?

7. $\dfrac{\frac{x+1}{x-2}-\frac{x}{x+2}}{\frac{x-3}{x^2-4}+\frac{1}{x+2}}$

8. $\dfrac{\frac{3}{x}-\frac{3}{2x+3}}{\frac{3}{x^2}+\frac{1}{2x+3}}$

EXTRA SPICY — Coordinate factoring, cancellation, and restrictions

9. $\dfrac{\frac{x-2}{x^2-4}+\frac{x+1}{x-2}}{\frac{x^2+3x+2}{2x^2+5x+2}-\frac{1}{2x+1}}$

10. In #9, a learner crosses out individual $x + 2$ factors in different fractions because they appear in both layers. Why is this cancellation invalid? Describe a valid first step instead.

11. $\dfrac{\frac{x^2-9}{x^2-4}-\frac{x+3}{x+2}}{\frac{x+2}{2x^2+5x+2}-\frac{1}{x+2}}$