Mixed Rational Expressions

Select and coordinate strategies to simplify rational expressions while preserving all NPVs.

Simplify → factor/cancel;
Multiply → factor/cancel/multiply;
Divide → restrictions/reciprocal;
Add/Subtract → LCD/combine/simplify;

State all original NPVs (including zero divisors).

PREVIOUSLY LEARNED — Choose the appropriate operation strategy

1. $\frac{x^2-4}{x^2+5x+6}$

2. $\frac{2x}{x+3} \div \frac{4x}{x-1}$

3. $\frac{2}{x-1} + \frac{3}{x+2}$

MEDIUM — Coordinate Two Operations

4. $\left(\frac{x^2-4}{x^2-1} \cdot \frac{x+1}{x-2}\right) + \frac{2}{x+2}$

5. $\left(\frac{x^2-1}{x^2+x-6} \div \frac{x+1}{x+3}\right) - \frac{1}{x+2}$

6. $\left(\frac{2}{x-1} + \frac{1}{x+1}\right) \cdot \frac{x^2-1}{3x+1}$

7. Consider: In question #6, two learners suggest different approaches:
   - Learner A wants to add the expressions inside the parentheses before multiplying.
   - Learner B wants to distribute the multiplication across both terms inside the parentheses.

   Both strategies are mathematically valid. Which approach would you choose, and why?

SPICY — More Demanding Combinations

8. $\left(\frac{x^2-4}{2x^2+5x+2} \div \frac{2-x}{x+1}\right) - \frac{1}{x+2}$

9. $\left(\frac{2}{(x-1)\text{^}2} - \frac{1}{x^2-1}\right) \div \frac{x+3}{(x-1)(x+2)}$

10. After simplifying question #9, a learner obtains $\frac{x+2}{(x-1)(x+1)}$. They conclude that the only NPVs are $x = -1$ and $x = 1$. Explain why their answer is incomplete. Identify the additional NPVs and where they originated.

11. $\left(\frac{x^2-9}{x^2-4} \div \frac{3-x}{x+2}\right) + \left(\frac{x-1}{x+1} \cdot \frac{2}{x-1}\right)$

EXTRA SPICY — Synthesis Challenge

12. $\left(\frac{2}{x-2} + \frac{3}{x+2}\right) \div \frac{5x-2}{2x^2+3x-2} - \frac{x}{2-x}$