Types of Factoring / Recognizing Structures

Before you factor — decide what you see

1. Look for a greatest common factor first.
2. Count and examine the terms.
3. Check for special squares, an ordinary trinomial, or a repeated object.
4. Choose, factor, and check.

Section A — Greatest Common Factor

For 1–4, factor out the greatest common factor. For 5–7, continue factoring where possible.

MILD	MEDIUM	SPICY	EXTRA SPICY
1. $6x + 9$	3. $12x^2 - 18x$	5. $8x^2 - 32$	7. $-3x^2 + 12x - 12$
2. $8x^2 + 12x$	4. $6x^2 + 9x + 12$	6. $2x^2 + 10x + 12$	

8. A learner begins factoring $-12x^2 + 72x - 108$ by writing $-4(3x^2-18x + 27)$. Explain why this does not use the greatest common factor, and give a better first step. Identify the remaining structure.

Section B — Trinomials, a = 1

Factor each expression.

MILD	MEDIUM	SPICY	EXTRA SPICY
9. $x^2 + 5x + 6$	11. $x^2 - 7x + 12$	13. $3x^2 + 15x + 18$	15. $24 + 2x - 2x^2$
10. $x^2 + 2x - 8$	12. $x^2 - x - 12$	14. $-2x^2 - 2x + 24$	

16. A learner decides to use non-monic trinomial factoring immediately on $6x^2 + 6x - 120$. Explain the first step they overlooked, then factor the expression.

Section C — Trinomials, a ≠ 1

Factor each expression. For 24, evaluate the learner's classification.

MILD	MEDIUM	SPICY	EXTRA SPICY
17. $2x^2 + 5x + 2$	19. $4x^2 - 8x + 3$	21. $6x^2 + 15x + 6$	23. $12x^2 + 10x - 12$
18. $3x^2 + 5x - 2$	20. $6x^2 + x - 2$	22. $-8x^2 + 4x + 12$	

24. A learner claims that $18x^2 + 21x - 9$ becomes a trinomial with a=1 after removing the GCF. Check the claim, identify the correct factoring type, and factor the expression.

Section D — Perfect-Square Trinomials
Factor each expression. For 31–32, analyze the learner's reasoning.

MILD

25. $x^2 + 6x + 9$

26. $x^2 - 10x + 25$

MEDIUM

27. $4x^2 + 12x + 9$

28. $9x^2 - 12x + 4$

SPICY

29. $3x^2 + 18x + 27$

30. $8x^2 - 24x + 18$

EXTRA SPICY

31. A learner classifies $25 + 30x + 9x^2$ as an ordinary non-monic trinomial. Explain why the perfect-square classification is more revealing, then factor it.

32. A learner factors $-2x^2 + 12x - 18$ as $-2(x+3)^2$. Identify the error and correct the factorization.

Section E — Difference of Squares
Factor each expression.

MILD

33. $x^2 - 16$

34. $4x^2 - 9$

MEDIUM

35. $x^2 - 9y^2$

36. $9x^2 - 16y^2$

SPICY

37. $5x^2 - 45$

38. $18x^2 - 8$

EXTRA SPICY

39. $(x + 2)^2 - 25$

40. $(2x - 1)^2 - (x + 3)^2$

Section F — Quadratic by Structure
Identify the repeated object, temporarily replace it with u, and factor the quadratic structure. Substitute the original expression back afterward.

MILD

41. $(x + 1)^2 + 5(x + 1) + 6$

42. $(x - 2)^2 - (x - 2) - 6$

MEDIUM

43. $2(x + 2)^2 + 7(x + 2) + 3$

44. $3(x - 2)^2 + 11(x - 2) + 6$

SPICY

45. $(x^2)^2 - 5(x^2) + 4$

46. $2(x^2)^2 - 7(x^2) + 3$

EXTRA SPICY

47. $(x^2 - 3x + 1)^2 - 5(x^2 - 3x + 1) + 6$

48. $6(x^2 - 4x + 4)^2 + (x^2 - 4x + 4) - 1$