Perpendicular Lines

Mild — Introducing Perpendicular Lines

Remember: Perpendicular lines meet at a right angle. Their slopes are negative reciprocals. For example, a slope of $\frac{2}{3}$ has a perpendicular slope of $-\frac{3}{2}$.

1. Consider the equations $y = 2x + 1$ and $y = -\frac{1}{2}x + 1$.
   a) Identify the slope and y-intercept of each line.
   b) Sketch both lines on the same coordinate plane.
   c) Describe the angle formed where the lines intersect.
   d) Compare the rise and run of each line.
   e) Multiply the two slopes. What do you notice?

2. A line has the equation $y = 5x - 2$.
   a) Identify the slope and y-intercept.
   b) Sketch the original line.
   c) Determine the slope of a line perpendicular to this line.
   d) Draw a perpendicular line passing through $(0,1)$.
   e) Determine the equation of the perpendicular line in the form $y = mx + b$.

3. A student claims that a line perpendicular to a line with slope $m = -2$ must have slope $m = 2$.
   a) Explain the student's error.
   b) Determine the correct perpendicular slope.

4. A line passes through $(-2,-1)$ with slope $m = -2$.
   a) Sketch the line using the coordinate and slope provided.
   b) Write the equation in point-slope form $y - y_1 = m(x - x_1)$.
   c) Algebraically rearrange the equation into $y = mx + b$.
   d) Determine the slope of a perpendicular line.
   e) Draw a perpendicular line passing through $(0,2)$.
   f) Determine the equation of the perpendicular line in the form $y = mx + b$.

Medium

5. A line passes through $(3,-4)$ with slope $m = \frac{2}{3}$.
   a) Sketch the line using the coordinate and slope provided.
   b) Write the equation in point-slope form.
   c) Algebraically rearrange the equation into $y = mx + b$.
   d) Determine the slope of a perpendicular line.
   e) Draw a perpendicular line passing through $(-2,1)$.
   f) Write the equation of the perpendicular line in point-slope form $y - y_1 = m(x - x_1)$.
   g) Algebraically rearrange the perpendicular equation into $y = mx + b$.

6. A line has the equation $y = -\frac{1}{4}x + 2$.
   a) Identify the slope and y-intercept.
   b) Sketch the line.
   c) Determine the slope of a perpendicular line.
   d) Draw a perpendicular line passing through $(2,3)$.
   e) Write the equation of the perpendicular line in point-slope form $y - y_1 = m(x - x_1)$.
   f) Algebraically rearrange the perpendicular equation into $y = mx + b$.

Spicy

7. A line passes through $(-4,3)$ and $(2,-5)$.
   a) Sketch the original line using the two coordinates provided.
   b) Determine the slope using rise and run.
   c) Write the equation in point-slope form $y - y_1 = m(x - x_1)$.
   d) Algebraically rearrange the equation into $y = mx + b$.
   e) Determine the slope of a perpendicular line.
   f) Draw a perpendicular line passing through $(4,1)$.
   g) Determine the equation of the perpendicular line in the form $y = mx + b$.

8. Consider the two lines $y = -3x + 6$ and $3y = x + 6$.
   a) Rearrange the second equation into the form $y = mx + b$.
   b) Identify the slope of each line.
   c) Determine whether the lines are parallel, perpendicular, or neither.
   d) Explain your reasoning using the relationship between their slopes.

Extra Spicy

9. A horizontal line has the equation $y = 3$.
   a) Sketch the horizontal line.
   b) Draw a perpendicular line passing through $(-2,5)$.
   c) Determine the equation of the perpendicular line.
   d) Explain why its equation cannot be written in the form $y = mx + b$.
   e) Is the perpendicular line a function of x? Explain.

10. A line passes through $(-4,4)$ and $(4,-2)$.
   a) Sketch the original line using the two coordinates provided.
   b) Determine the slope using rise and run.
   c) Determine the equation of the original line in the form $y = mx + b$.
   d) Draw a line parallel to the original that passes through $(0,-3)$.
   e) Draw a line perpendicular to the original that also passes through $(0,-3)$.
   f) Determine the equation of each new line in the form $y = mx + b$.
   g) Explain how the slopes and graphs confirm which new line is parallel and which is perpendicular.