Trigonometric Functions & Proofs Review (Part 1)

1. **(Medium)** Determine whether the following pairs of angles are **coterminal**. If they are coterminal, write a **general formula** for all coterminal angles.

   a) $\frac{7\pi}{4}$ and $\frac{23\pi}{4}$

   b) $-\frac{11\pi}{6}$ and $\frac{13\pi}{6}$

2. **(Spicy)** Suppose angles A and B are **coterminal**. Determine whether each statement is **Always True**, **Sometimes True**, or **Never True**. Justify your reasoning.

   a) $\sin A = \sin B$
   b) $\tan A = \tan B$
   c) $A - B = 2\pi n$, where n is an integer

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3. **(Medium)** Determine the **exact value** without a calculator.

   a) $\sin\left(\frac{5\pi}{6}\right)$

   b) $\cos\left(\frac{7\pi}{4}\right)$

   c) $\tan\left(\frac{11\pi}{6}\right)$

4. **(Spicy)** Suppose $\sin\theta = -\frac{5}{13}$. Determine:

   a) **all possible coordinates** on the unit circle
   b) **all possible values** of $\cos\theta$
   c) **all possible values** of $\tan\theta$
   d) the **possible quadrants**

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5. **(Medium)** Determine the **exact value**.

   a) $\csc\left(\frac{\pi}{6}\right)$

   b) $\sec\left(\frac{2\pi}{3}\right)$

   c) $\cot\left(\frac{5\pi}{4}\right)$

6. **(Spicy)** Determine whether each equation is **possible**. Explain your reasoning.

   a) $\csc\theta = -3$

   b) $\sin\theta = 1.2$

   c) $\cot\theta = 0$

   d) $\sec\theta = 0.5$

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7. **(Medium)** Solve on the interval $0 \leq \theta < 2\pi$

   a) $\sin\theta = \frac{1}{2}$

   b) $\cos\theta = -\frac{\sqrt{2}}{2}$

8. **(Spicy)** Solve on the interval $0 \leq \theta < 2\pi$

$$3\sin^2\theta + \sin\theta - 2 = 0$$

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9. **(Medium)** Solve on the interval **0° ≤ θ < 360°**.

$$3\sin\ \theta = 2$$

Round answers to the **nearest tenth of a degree**.

10. **(Spicy)** Determine whether the equation

$$\sec\ \theta = 0.8$$

has a solution on **0° ≤ θ < 360°**. If a solution exists, solve. If no solution exists, explain why.

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11. **(Medium)** For $y = -\cos(2x)$ determine:
    a) the **amplitude**
    b) the **period**
    c) the **quarter-period**
    d) the **five key points** for one complete cycle

12. **(Spicy)** Compare the graphs $y = \sin(2x)$ and $y = -\cos\ (2x)$
    a) Compare the **amplitudes**.
    b) Compare the **periods**.
    c) Compare the **starting behaviour**.
    d) Compare the **number of cycles** on $0 \leq x \leq 2\pi$.
    e) Describe the **transformations** from the parent functions.
    f) **Sketch both graphs on the same set of axes.**

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13. **(Medium)** Determine all values of $x$on the interval $0 < x < 2\pi$ for which

$$\sin\ x = \cos\ x$$

State the **coordinates of the intersection points**.

14. **(Spicy)** Suppose the graphs $y = \sin\ x$ and $y = k$ intersect **exactly once** on the interval $0 \leq x \leq 2\pi$**.** Determine all possible values of $k$and justify your reasoning.

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15. **(Medium)** Without graphing, determine the **maximum value**, **minimum value**, **range**, and **midline** for

$$y = -4\sin\ (x - \frac{\pi}{3}) + 2$$

16. **(Spicy)** Explain why $y = 2\sin x + 5$ has **no x-intercepts**. Use the **range**, **amplitude**, and **midline** in your explanation.

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17. **(Medium)** For $y = 3\cos\left(x - \frac{\pi}{2}\right) + 1$ determine:
    a) the **first key x-value**
    b) the **last key x-value**
    c) the **quarter-period**
    d) the **five key points** for one complete cycle

18. **(Spicy)** A trigonometric graph has **amplitude 5**, **midline** $y = -2$ and **phase shift** $\frac{\pi}{4}$**right**. Determine:
    a) sketch the graph
    b) the **five key points** for one complete cycle of a possible sine function

Trigonometric Functions & Proofs Review (Part 2)

19. **(Medium)** Determine the **period** and **number of cycles** on $\mathbf{0 \leq \theta \leq 2\pi}$.
    a) $y = \sin(3\theta)$
    b) $y = \cos(\frac{\theta}{2})$

20. **(Spicy)** Determine the **number of x-intercepts** of $y = \sin(5\theta)$ on $\mathbf{0 \leq \theta \leq 2\pi}$. Explain your reasoning using:
    - **period**
    - **number of cycles**
    - **properties of sine graphs**

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21. **(Medium)** Determine whether the function $y = 3\sin(2\theta) + 5$ has any **x-intercepts**. Justify without solving algebraically.

22. **(Spicy)** Without graphing, determine the **amplitude**, **period**, **number of cycles** on $\mathbf{0 \leq \theta \leq 2\pi}$, **maximum value**, **minimum value**, **range**, and **all the x-intercepts** for

$$y = -4\sin(3\theta - \frac{\pi}{2}) + 2$$

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23. **(Medium)** Write a **cosine function** with:
- **maximum** value 8
- **minimum** value 2
- **phase shift** $\frac{\pi}{3}$ right

24. **(Spicy)** Design **two different trigonometric equations** that produce **exactly the same graph**. Explain why they are equivalent using **transformations**.

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25. **(Medium)** A city receives **15 hours of daylight** at its maximum and **9 hours of daylight** at its minimum. Determine:
    a) the **amplitude**
    b) the **midline**

26. **(Spicy)** A Ferris wheel has:
    - **maximum height 54 m**
    - **minimum height 6 m**
    - **period 20 minutes**

    A rider starts at the **lowest point**.
    a) Create a **trigonometric model**.
    b) Determine **all times during the first revolution** when the rider is above **40 m**.

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27. **(Medium)** Identify the **identity family** that would be most useful.
    a) $\sec^2\theta - 1$
    b) $1 - \sin^2\theta$
    c) $\tan\theta \cdot \cot\theta$

28. **(Spicy)** Simplify completely.

$$\frac{\tan\theta+\cot\theta}{\sec\theta\cdot\csc\theta}$$

Show all steps and identify the identities used.

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29. **(Medium)** Prove algebraically:

$$(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)=1$$

30. **(Spicy)** Prove algebraically:

$$\sec^2 x+\csc^2 x=\sec^2 x\csc^2 x$$

Your proof should be fully justified and simplified.

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31. **(Medium)** Identify the **single trigonometric function** represented by:

$$\cos 70^\circ\cos 20^\circ+\sin 70^\circ\sin 20^\circ$$

32. (Spicy) Rewrite the angle as a **sum or difference of special angles** and determine the **exact value**:

$$\sin\left(\frac{5\pi}{12}\right)$$

Show all work.

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33. **(Medium)** Given $\sin\theta=\frac{3}{5}$ and θ is in **Quadrant I**, determine:

a) $\sin(2\theta)$
b) $\cos(2\theta)$

34. **(Spicy)**

a) Rewrite as a **single trigonometric function**: $2\sin(3x)\cos(3x)$
b) Rewrite as a **single trigonometric function**: $\cos^2(2x)-\sin^2(2x)$
c) Explain how the two answers are connected through the **double-angle identities**.