Advanced Double Angle Identities

Part A – Building New Forms

1. Use the double-angle identities to derive each result.

   a) $\sin(2\theta) = 2\cot\theta\sin^2\theta$

   b) $\cos(2\theta) = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

   c) $\sec^2\theta = \frac{2}{1+\cos(2\theta)}$

   d) $\frac{1-\cos(2\theta)}{2} = \sin^2\theta$

Part B – Solving Equations

2. Solve for $0 \le \theta < 2\pi$.

   a) $\sin\left(\theta+\frac{\pi}{2}\right) = \frac{1}{2}$

   b) $\cos\left(\frac{3\pi}{2}+\theta\right) + \sin\left(\frac{3\pi}{2}-\theta\right) = 0$

   c) $\sin\left(\theta+\frac{\pi}{3}\right) + \sin\left(\theta-\frac{\pi}{3}\right) = 1$

Part C – Introductory Double-Angle Proofs

3. Prove each identity.

   a) $1 + \sin(2\theta) = (\sin\theta + \cos\theta)^2$

   b) $\frac{1-\cos(2\theta)}{2} = \sin^2\theta$

   c) $\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta} = -\sec(2\theta)$

   d) $\cos^4 x - \sin^4 x = \cos(2x)$

Part D – Extending the Identities

4. Prove each identity.

   a) $\cot(2\theta) = \frac{1}{2}(\cot\theta - \tan\theta)$

   b) $\frac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta} = \cos(2\theta)$

   c) $\cos^2(2\theta) - \sin^2(2\theta) = \cos(4\theta)$

Part E – Capstone Problems

5. Prove each identity.

   a) $(4\sin\theta\cos\theta)(1 - 2\sin^2\theta) = \sin(4\theta)$

   b) $\sin(2\theta) = 2\tan\theta - 2\tan\theta\sin^2\theta$

   c) $3\sin^2\theta\cos^2\theta = \frac{3}{8}(1 - \cos 4\theta)$