

Applications of Sum and Difference Identities

Part A – Rewrite as a Single Trigonometric Function

1. Rewrite each expression as a single trigonometric function.

a) $\cos 70^\circ \cos 20^\circ + \sin 70^\circ \sin 20^\circ$

d) $\cos 3x \cos x + \sin 3x \sin x$

b) $\sin 20^\circ \cos 10^\circ + \cos 20^\circ \sin 10^\circ$

e) $\sin 2x \cos x - \cos 2x \sin x$

c) $\cos 110^\circ \cos 10^\circ - \sin 110^\circ \sin 10^\circ$

f) $\frac{\tan(\pi/9) + \tan(5\pi/36)}{1 - \tan(\pi/9)\tan(5\pi/36)}$

Part B – Variable-Angle Simplification

2. Simplify each expression.

a) $\cos(\theta + 2\pi)$

e) $\sin\left(\theta - \frac{3\pi}{2}\right)$

b) $\sin(\theta + \pi)$

f) $\sin\left(\frac{\pi}{3} - x\right) \cos\left(\frac{\pi}{3} + x\right) + \cos\left(\frac{\pi}{3} - x\right) \sin\left(\frac{\pi}{3} + x\right)$

c) $\sin\left(\theta + \frac{\pi}{2}\right)$

g) $\sin a \cos 2a + \cos a \sin 2a$

d) $\tan\left(\theta + \frac{\pi}{2}\right)$

h) $\cos\left(x + \frac{\pi}{2}\right) - \cos\left(x - \frac{\pi}{2}\right)$

Part C – Given-Ratio Applications

3. For this question, assume the angles are in Quadrant I. Given $\sin a = \frac{\sqrt{3}}{2}$ and $\cos b = \frac{1}{\sqrt{2}}$, find:

a) $\sin(a + b)$

c) $\sin(a - b)$

b) $\cos(a + b)$

d) $\cos(a - b)$

Part D – Challenge Problems

4. Given $\sin(a + b) = \frac{1}{2}$ and $\cos(a + b) = \frac{\sqrt{3}}{2}$ find: $\tan(a + b)$

5. Given $\sin(a + b) = \frac{3}{5}$ and $a + b$ lies in Quadrant I, find: $\cos(a + b)$

6. Suppose $\cos x = 0$ and $\cos(x + z) = 0.5$. Find the smallest positive value of z .

7. Suppose $\sin a + \sin b = \frac{\sqrt{5}}{3}$ and $\cos a + \cos b = 1$. Find: $\cos(a - b)$