

Proving and Trigonometric Identities

When proving a trigonometric identity:

- Work on both sides, when possible.
- Switch back and forth between sides.
- Convert $\tan \theta$, $\cot \theta$, $\sec \theta$, and $\csc \theta$ to sine and cosine whenever helpful.
- Look for opportunities to use:
 - $\sin^2 \theta + \cos^2 \theta = 1$
 - $1 + \tan^2 \theta = \sec^2 \theta$
 - $1 + \cot^2 \theta = \csc^2 \theta$
- Simplify completely.

Part A – Direct Identity Substitutions

Prove each identity algebraically.

1. $(\sec \theta - \tan \theta)(\sec \theta + \tan \theta) = 1$

2. $\frac{\sec^2 \theta - 1}{\sec^2 \theta} = \sin^2 \theta$

3. $(\tan^2 \theta + 1)(\cos^2 \theta - 1) = -\tan^2 \theta$

Part B – Convert Everything to Sine and Cosine

Prove each identity algebraically.

4. $\tan \theta + \cot \theta = \sec \theta \csc \theta$

7. $\sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$

5. $\frac{\sec \theta}{\tan \theta + \cot \theta} = \sin \theta$

8. $\frac{\sin \theta + \tan \theta}{\cos \theta + 1} = \tan \theta$

6. $\frac{\csc \theta - \sin \theta}{\cot \theta} = \cos \theta$

Part C – Adding Rational Expressions

Prove each identity algebraically.

9. $\frac{1}{1+\cos \theta} + \frac{1}{1-\cos \theta} = \frac{2}{\sin^2 \theta}$

10. $\frac{1}{1-\sin \theta} + \frac{1}{1+\sin \theta} = 2\sec^2 \theta$

11. $\frac{\sin \theta}{1+\cos \theta} + \frac{1+\cos \theta}{\sin \theta} = 2\csc \theta$

Part D – Using Conjugates

If a denominator contains a binomial such as $1 - \sin \theta$ or $1 - \cos \theta$, consider multiplying by the conjugate. Prove each identity algebraically.

$$12. \frac{1 + \sin \theta}{\cos \theta} = \frac{\cos \theta}{1 - \sin \theta}$$

$$15. \frac{1 - \cos \theta}{\sin \theta} = \frac{1}{\csc \theta + \cot \theta}$$

$$13. \sec \theta + \tan \theta = \frac{1}{\sec \theta - \tan \theta}$$

$$16. \frac{\sec \theta}{1 - \cos \theta} = \frac{\sec \theta + 1}{\sin^2 \theta}$$

$$14. \frac{\csc \theta - 1}{\cot \theta} = \frac{\cot \theta}{\csc \theta + 1}$$