16 - Applications & Modelling with Trigonometric Functions

1. The London Eye is a large Ferris wheel with a height of 135m. Passengers board the attraction in one of the 32 capsules on a platform 15m above ground. One revolution takes about 30 minutes.

   a) Write a sine or cosine function for the height of a passenger above ground (meters) as a function of time (minutes) after boarding the ride.
   b) How high above ground will a passenger be after 13 minutes?
   c) After how many minutes after boarding the ride will a passenger reach a height of 100m above ground?
   d) When a passenger is near the top, at a height of at least 120m, they can see a building that is 40km away. For how many minutes on the ride can they see this building?

2. During high tide, the depth of water levels at the coast is around 4.8m deep. During low tide, the water levels can drop to 0.5m deep. "Low Low" tide on Jan 4, 2023 occurs at 10:30pm and the next "Low Low" tide occurs on Jan 5, 2023 at 11:00pm.

   a) Write a sine or cosine function for the depth of water levels (meters) as a function of time (minutes) after 10:30pm on Jan 4, 2023.
   b) What is the depth of water levels at 4:00am on Jan 5, 2023?
   c) At what time(s) is the water depth exactly 3.0m?
   d) Boating and fishing are best during high tide. Suppose you want to go fishing when tide levels are at least 4m. What time should you go boating?

3. The Earth is closest to the Sun on December 21 at approximately 147.2 million km. On June 21, the Earth is farthest from the Sun at approximately 152.2 million km.

   a) Write a sine or cosine function for the distance D(million km) from the Earth to the Sun as a function of day number T.
   b) Use your function to estimate the distance between the Earth and Sun on February 28.
   c) On what day(s) is the Earth exactly 151 million km from the Sun?
   d) On which days is the distance between the Earth and Sun at least 150 million km?

4. A city receives approximately 16 hours of daylight on June 21 and 8 hours of daylight on December 21. Assume daylight hours vary sinusoidally throughout the year.

   a) Write a trigonometric model for the number of daylight hours Has a function of day number T, where $T = 0$ corresponds to January 1.
   b) Use your model to estimate the number of daylight hours on September 21.
   c) On what day(s) does the city receive exactly 14 hours of daylight?
   d) During what part of the year does the city receive at least 14 hours of daylight?

Consider the general trigonometric sine and cosine functions:

$$y = a\,sin(b(x - c)) + d$$

$$y = a\,cos(b(x - c)) + d$$

For each graph, find the midline, amplitude, phase shift, and period. Then, create two functions that match the graph, one sine and one cosine function.