15 — Function Design & Synthesis

Part I — Building Equations

1. A trigonometric graph has the following maximum and minimum values. Determine the amplitude and midline.
   a) Maximum = 6, Minimum = 2
   b) Maximum = 10, Minimum = -4
   c) Maximum = 8, Minimum = -2

2. Write a sine function with a maximum value of 6 and a minimum value of 2.

3. Write a cosine function with a maximum value of 10 and a minimum value of -4.

4. Write a sine function with amplitude 3 and phase shift $\frac{\pi}{2}$ right.

5. Write a cosine function with amplitude 5 and phase shift $\frac{\pi}{3}$ left.

Part II — Equivalent Forms

6. Write a cosine function equivalent to $y = 3\sin\ x$.

7. Write a sine function equivalent to $y = 4\cos\ x$.

8. Rewrite each function as an equivalent function of the requested type.
   a) Write a cosine function equivalent to $y = -2\sin\ x$
   b) Write a sine function equivalent to $y = 5\cos\ x$
   c) Write a cosine function equivalent to $y = 4\sin\ (2x)$

Part III — Constructing From Conditions

9. Find an equation in the form $y = a\sin\ (x - b) + d$ with maximum value 3 and minimum value -2.

10. Find an equation in the form $y = a\cos\ (x - c) + d$ with maximum value 8, minimum value -2, and phase shift $\frac{\pi}{2}$right.

11. A trigonometric function has maximum value 12, minimum value 4, and period 2π. Write:
    a) a possible sine equation
    b) a possible cosine equation

12. A trigonometric function has maximum value 10, minimum value 4, and maximum occurs at $x = 0$. Write a possible cosine equation.

13. A wave has a maximum point at $\left(\frac{\pi}{3}, 10\right)$ and the next minimum point at $\left(\frac{4\pi}{3}, 4\right)$. Determine a possible trigonometric equation.

Summary Problems

14. A trigonometric graph has amplitude 4, midline $y = 2$, period π, and phase shift $\frac{\pi}{4}$ right. Write an equation for the graph and justify the amplitude, vertical shift, period, and phase shift.

15. A trigonometric graph has maximum value 7, minimum value -1, period $\frac{2\pi}{3}$, and phase shift $\frac{\pi}{6}$ left. Write:
    a) a sine equation
    b) a cosine equation

16. Design a trigonometric function that satisfies all of the following conditions:
    - amplitude 6
    - midline $y = -2$
    - period 4π
    - phase shift $\frac{\pi}{3}$right
    - minimum value occurs at $x = \frac{\pi}{3}$

    Justify every part of your equation.

Proof and Equivalence

17. Two of the following functions are equivalent. Identify the pair and justify your answer using transformations.

    i. $y = 3\sin\left(2\left(x+\frac{\pi}{2}\right)\right)$
    ii. $y = 3\cos(2x)$
    iii. $y = 3\cos\left(2\left(x+\frac{\pi}{4}\right)\right)$
    iv. $y = 3\sin(2(x+\pi))$

18. Two of the following functions are equivalent. Identify the pair and prove your answer algebraically.
    i. $y = 3\sin(2(2x+\pi))$
    ii. $y = 3\cos\left(2x+\frac{\pi}{2}\right)$
    iii. $y = 3\cos(2x)$
    iv. $y = 3\sin(2(2x+2\pi))$

19. Design two different trigonometric equations that produce the same graph. Explain why they are equivalent.