

14 — Structural Analysis of Trigonometric Functions

Part I — Structural Analysis

1. For each function, determine the amplitude, period, number of cycles between $0 \leq \theta \leq 2\pi$, maximum value, minimum value, and range.

a) $y = -4\sin(3\theta) + 2$

b) $y = 5\cos\left(\frac{\theta}{2}\right) - 1$

2. Determine whether each function has any x-intercepts. Justify your answer without solving.

a) $y = 2\sin(3\theta) + 5$

b) $y = -4\cos(2\theta) + 1$

c) $y = 3\sin\left(\frac{\theta}{2}\right) - 7$

Part II — X-Intercept Analysis

3. Without solving exactly, determine how many x-intercepts exist on $0 \leq \theta \leq 2\pi$ for $y = 3\sin(2\theta) + 1$. Explain your reasoning.
4. Determine all x-intercepts of $y = 3\sin(2\theta) + 1$ on $0 \leq \theta \leq 2\pi$. Show all work.
5. Determine all x-intercepts of $y = -5\cos\left(\frac{\theta}{2}\right) + 2$ on $0 \leq \theta \leq 2\pi$. Show all work.

Part III — Reflections

6. Compare the following functions. Describe the transformation, the effect on the graph, and any similarities you notice.

$$y = -\sin \theta$$

$$y = \sin(-\theta)$$

7. Explain why a vertical reflection of a sine function is equivalent to a horizontal reflection of a sine function.
8. What is special about a horizontal reflection of a cosine function?

Part IV — Equivalent Functions

9. Determine whether each pair of functions represents the same graph. Justify using transformations.

a) $y = \sin \theta$ and $y = \cos\left(\theta - \frac{\pi}{2}\right)$

b) $y = -\sin \theta$ and $y = \sin(\theta + \pi)$

c) $y = \cos \theta$ and $y = \sin\left(\theta + \frac{\pi}{2}\right)$

10. Find a simpler equivalent expression for each function using trigonometric transformations. State the transformation that justifies your answer.

a) $\sin(x + 2\pi)$

c) $\sin(x + \pi)$

b) $\cos(x + 2\pi)$

d) $\cos(x + \pi)$

11. Two of the following functions are equivalent. Identify the pair and justify your answer using transformations.

i. $y = 3\sin\left(2\left(\theta + \frac{\pi}{2}\right)\right)$

iii. $y = 3\cos\left(2\left(\theta + \frac{\pi}{4}\right)\right)$

ii. $y = 3\cos(2\theta)$

iv. $y = 3\sin(2(\theta + \pi))$

12. Two of the following functions are equivalent. Identify the pair and prove your answer algebraically.

i. $y = 3\sin(2(2\theta + \pi))$

iii. $y = 3\cos(2\theta)$

ii. $y = 3\cos\left(2\theta + \frac{\pi}{2}\right)$

iv. $y = 3\sin(2(2\theta + 2\pi))$

Part V — Solving for B

13. Determine the value of B if the period of $y = 5\sin(B(\theta - 2)) + 7$ is:

a) π

b) 4

c) 7

d) $\frac{\pi}{3}$

14. Determine the value of B for each function.

a) $y = \sin(B\theta)$ has 3 complete cycles on $0 \leq \theta \leq 2\pi$

b) $y = \cos(B\theta)$ has 4 complete cycles on $0 \leq \theta \leq 2\pi$

c) $y = \sin(B\theta)$ has period $\frac{\pi}{2}$

d) $y = \cos(B\theta)$ has period 3

Challenge Problems

15. Without graphing, determine amplitude, period, number of cycles on $0 \leq \theta \leq 2\pi$, maximum value, minimum value, range, all x-intercepts for $y = -4\sin\left(3\theta - \frac{\pi}{2}\right) + 2$

16. Prove that the following functions produce the same graph $y = -\sin \theta$ and $y = \sin(\theta + \pi)$. Provide a transformation argument