

13 — Period Changes and the B Value

Part I — Understanding the B Value

1. Compare the following three functions:

$$y = \sin \theta$$

$$y = \sin (2\theta)$$

$$y = \sin \left(\frac{\theta}{2} \right)$$

For each function:

- Determine the period.
 - Determine the number of cycles between $0 \leq \theta \leq 2\pi$.
 - Identify which graph is the most compressed.
 - Identify which graph is the most stretched.
2. Explain how the value of B affects:
- the period
 - the number of cycles
 - the spacing between key points
 - the appearance of the graph

Part II — Calculating Periods

3. Determine the period of each function.

a) $y = 2\cos \left(\frac{\theta}{3} \right) + 5$

c) $y = 5\sin \left(\frac{2}{3}(\theta - 2) \right) + 7$

b) $y = 3\cos (4\theta) - 1$

d) $y = 5\sin \left(\frac{2\pi}{3}(\theta - 2) \right) + 7$

4. Determine the number of complete cycles between $0 \leq \theta \leq 2\pi$.

a) $y = \cos (2\theta)$

c) $y = \cos \left(\frac{\theta}{2} \right)$

b) $y = \cos (3\theta)$

d) $y = \cos \left(\frac{\theta}{4} \right)$

Part III — Intercepts and Cycles

5. Without graphing, determine the number of x-intercepts between $0 \leq \theta \leq 2\pi$.

a) $y = \sin \theta$

c) $y = \sin (3\theta)$

b) $y = \sin (2\theta)$

d) $y = \sin (4\theta)$

6. For sine functions on $0 \leq \theta \leq 2\pi$, explain the relationship between:

- the value of B
- the number of cycles
- the number of x-intercepts

Part IV — Building Graphs with B

7. For the function $y = 3\sin(2\theta) + 1$ determine the quarter-period and the five key coordinates for one complete cycle.
8. Graph $y = 3\sin(2\theta) + 1$ and then state all transformations.

Part V — Stretching the Graph

9. For the function $y = 4\cos\left(\frac{\theta}{2}\right) + 2$, determine the quarter-period and the five key coordinates for one complete cycle.
10. Graph $y = 4\cos\left(\frac{\theta}{2}\right) + 2$. State all transformations.

Part VI — Mixed Transformations

11. Determine the quarter-period and the five key coordinates for one complete cycle of $y = 3\sin\left(\frac{2\pi}{3}(\theta - 2)\right) + 1$.
12. Graph $y = 3\sin\left(\frac{2\pi}{3}(\theta - 2)\right) + 1$. State all transformations.

Challenge Problems

13. Determine the period, the number of cycles between $0 \leq \theta \leq 2\pi$, the maximum value, and the minimum value of $y = -5\cos(4\theta) + 2$.
14. Predict whether the graph of $y = -5\cos(4\theta) + 2$ will have more x-intercepts, fewer x-intercepts, or the same number of x-intercepts as $y = -5\cos(2\theta) + 2$. Explain your reasoning.