Graphing Sine & Cosine Functions

Part I — Unit Circle Connections

1. On the unit circle:
   a) What does the sine function represent?
   b) What does the cosine function represent?
   c) Explain why the parent functions $y = \sin\ \theta$and $y = \cos\ \theta$ both have an amplitude of 1.
   d) Explain why both functions repeat every 2π radians.

Part II — Graphing Parent Functions

2. For the function $y = \sin\theta$.
   a) Determine amplitude, period, domain, range
   b) Divide the period into four equal parts to determine the quarter-period.
   c) Use quarter-periods to determine the five key x-values needed to graph one complete cycle.
   d) Determine the corresponding y-values.
   e) Sketch one complete cycle on a Cartesian plane.
   f) Explain why quarter-periods are useful when graphing trigonometric functions.

3. For the function $y = \cos\ \theta$
   a) Determine the amplitude, period, domain, range.
   b) Use quarter-periods to determine the five key points for one complete cycle.
   c) Identify x-intercepts, y-intercept, maximum values, minimum values
   d) Sketch one complete cycle on a Cartesian plane.
   e) Compare the starting point of the cosine graph to the sine graph.

4. Compare the graphs of $y = \sin\ \theta$ and $y = \cos\ \theta$
   a) Describe the starting points, maximum values, minimum values
   b) Sketch both graphs on the same Cartesian plane.
   c) Describe how the graphs are similar and different.

Part III — Reflections

5. Graph the following functions on the same Cartesian plane: $y = \sin\theta$ and $y = -\sin\ \theta$
   a) Describe how the graph changes.
   b) Identify the amplitude, period, domain, range.
   c) Determine the 5 key points for one complete cycle of $y = -\sin\ \theta$.
   d) Sketch both graphs on the same Cartesian plane.
   e) Explain how one graph can be transformed into the other.

6. Compare the graphs of $y = \cos\theta$ and $y = -\cos\theta$
   a) Describe how the graph changes.
   b) Label the x-intercepts, y-intercept, maximum values, minimum values.
   c) Determine the five key points for one complete cycle of $y = -\cos\ \theta$
   d) Sketch both graphs on the same Cartesian plane.
   e) Explain how one graph can be transformed into the other.

7. For the function $f(\theta) = \cos(0.5\theta)$
   a) Determine the amplitude,
   b) Use the formula Period $= \frac{2\pi}{|b|}$ to determine the period.
   c) Determine the quarter-period.
   d) Use quarter-periods to determine the five key points for one complete cycle.
   e) Sketch one complete cycle on a Cartesian plane.
   f) Compare the graph to $y = \cos\ \theta$ and describe how the graph changed.

8. Compare the graphs of $y = \sin(0.5\theta)$ and $y = \cos(0.5\theta)$
   a) Compare starting points, periods, maximum values, minimum values
   b) Sketch both graphs on the same Cartesian plane.
   c) Explain why both graphs have the same period.

9. For the function $y = \cos(2\theta)$
   a) Determine the amplitude, period, quarter period
   b) Determine the 5 key points for one complete cycle.
   c) Sketch two complete cycles on a Cartesian plane.
   d) Compare the graph to $y = \cos\ \theta$ and describe how the graph changed.

10. Compare the graphs of $y = \sin\ \theta$ and $y = \sin(2\theta)$
    Compare the:
    - periods
    - number of cycles
    - Spacing of key points

    Sketch both graphs on the same Cartesian plane.

11. Compare the graphs of $y = \sin(2\theta)$ and $y = \cos(2\theta)$
    a) Compare periods, starting points, and number of cycles on $0 \leq \theta \leq 2\pi$
    b) Sketch both graphs on the same Cartesian plane.

12. Compare and graph $y = -\sin(2\theta)$and $y = -\cos(2\theta)$
    a) Describe reflections, horizontal compression, starting points
    b) Determine the period and quarter-period of each function.
    c) Sketch both graphs on the same Cartesian plane.
    d) Explain all transformations that occurred from the parent functions.

13. Compare the graphs of $y = \cos(0.5\theta)$ and $y = -\cos(0.5\theta)$
    a) Describe how the graph changed.
    b) Determine the period and quarter-period.
    c) Determine the five key points for one complete cycle.
    d) Sketch both graphs on the same Cartesian plane.
    e) Explain all transformations that occurred.