Continuously Compounded Investments

Annuity (end of year):

$$S_n = A\left(\frac{e^{rn}-1}{e^r-1}\right)$$

Annuity due (beginning of year):

$$S_n = Ae^r\left(\frac{e^{rn}-1}{e^r-1}\right)$$

Section A — Core Calculations

1. An engineer deposits \$900 at the **beginning** of each year into an account earning 6% continuously. Find the value after 7 years.

2. A worker deposits \$750 at the **beginning** of each year into an account earning 5% continuously. Find the value after 10 years.

3. Carlos deposits \$800 at the **beginning** of each year into an account earning 6% continuously. How many years will it take for the balance to reach \$15,000?

Section B — Solving for Unknowns

4. Oliver wants \$10,000 in 8 years. If the account earns 5% continuously and deposits are made at the **beginning** of each year, how much must be deposited annually?

5. An investor deposits \$1,200 at the **beginning** of each year at 5% continuous interest. How many years will it take to reach \$20,000?

Section C — Mixed Investments

6. Sophie has \$5,000 and adds \$1,500 at the **end** of each year into an account earning 5% continuously. Find the total after 10 years.

7. An investor starts with \$8,000 and deposits \$1,000 at the **beginning** of each year into an account earning 4% continuously. Find the total after 12 years.

8. A scholarship fund requires payments of \$1,200 at the **beginning** of each year for 10 years. The account earns 5% continuously. What lump sum must be invested today to cover these payments?

Section D — Reverse Problems

9. An account grows to \$18,000 after 10 years. Deposits of \$900 are made at the **beginning** of each year at 5% continuous interest. Find the initial lump sum invested.

10. An investor deposits \$1,200 at the **beginning** of each year. After 12 years, the account is worth \$25,000. Find the interest rate (continuous compounding).

11. How long will it take for deposits of \$1,000 at the **beginning** of each year to reach \$20,000 at 5% continuous interest?

Section E — Comparison

12. Compare the following plans over 12 years:
    - Plan A: \$600 deposited at the **end** of each year at 4% continuous interest
    - Plan B: \$1,000 deposited at the **end** of each year at 3% continuous interest

    Which plan yields a higher balance, and by how much?

13. Without fully calculating, determine which plan will produce the greater final value. Justify your reasoning.
    - Plan A: \$1,000/year at 4% (continuous)
    - Plan B: \$900/year at 5% (continuous)

Section F — Reasoning and Error Analysis

14. A student uses the formula

$$S_n = 1200e^{0.05}\left(\frac{e^{0.5}-1}{e^{0.05}-1}\right)$$

    to calculate the value of deposits made at the **end** of each year.
    - Identify the mistake
    - Write the correct formula
    - Explain the difference

15. A student claims: “Doubling the interest rate will double the final amount.” Using a continuously compounded annuity, explain why this statement is incorrect.