Exponents & Logarithms Review (Part 1)

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1. (Medium)

Consider the function $f(x) = 2 \cdot 3^{x-1} - 4$

(a) Determine the **y-intercept**.

(b) Describe the **end behavior** as:

- $x \to \infty$
- $x \to -\infty$

2. (Spicy) Consider the function $g(x) = -4 \cdot 2^{-(x+2)} + 6$

(a) Determine the **y-intercept**.

(b) Describe the **end behavior** as:

- $x \to \infty$
- $x \to -\infty$

(c) State whether the function represents **exponential growth or decay**, and justify your answer.

(d) Sketch the graph, showing all key features.

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3. (Medium)

A population of fish in a lake grows exponentially. Initially, there are **1200 fish**. After **5 years**, the population has grown to **3000 fish**.

(a) Determine an exponential model of the form $P(t) = P_0 \cdot r^t$

(b) Determine the population after **10 years**.

4. (Spicy)

A certain investment grows exponentially. An initial investment of **$5000** increases to **$13,200** after **8 years**.

(a) Determine an exponential model of the form $A(t) = A_0 \cdot r^t$

(b) How long will it take for the investment to reach **$25,000**?

(c) Determine the **annual growth rate as a percentage**.

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5. (Medium) The decibel level of a sound is given by $dB = 10\log_{10}\left(\frac{I}{I_0}\right)$. Sound A has a level of **60 dB** and Sound B has a level of **80 dB**.

(a) How many times more intense is Sound B than Sound A?

6. (Spicy)

The difference in decibel levels between two sounds is given by $dB_B - dB_A = 10\log_{10}\left(\frac{I_B}{I_A}\right)$. Sound A has a level of **72 dB**.

(a) A second sound is **250 times more intense** than Sound A. Determine the decibel level of this sound.

(b) A third sound has a level of **92 dB**. How many times more intense is this sound compared to Sound A?

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7. (Medium) Rewrite the following as a **single logarithm**:

$$\log_2(5x) + \log_2(3x^2) - \log_2(4)$$

8. (Spicy) Rewrite the following as a **single logarithm**, fully simplified:

$$3\log_5(2x^2) - \log_5(5x) + \frac{1}{2}\log_5(25x^3) - 2\log_5(4)$$

9. (Medium) Solve the equation:

$$\log_3(x) + \log_3(x-2) = 2$$

10. (Spicy) Solve the equation:

$$\log_2(x+3) + \log_2(x-1) = 3$$

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11. (Medium) Let $\log_a b = m$ and $\log_a c = n$

Express the following in terms of $m$and n:

$$\log_a(b^2c)$$

12. (Spicy) Let $\log_a b = m$ and $\log_a c = n$

Express the following in terms of $m$and n: $\log_a\left(\frac{b^3\sqrt{c}}{a^2c^4}\right)$

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13. (Medium) Solve the equation:

$$5 \cdot 2^x = 80$$

14. (Spicy) Solve the equation:

$$3^{2x-1} = 5^{x+2}$$

Give your answer as an **exact value in logarithmic form**.

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15. (Medium) A population of bacteria grows exponentially. Initially, there are **900 bacteria**. After **4 hours**, the population has grown to **2700 bacteria**.

(a) Determine an exponential model of the form $P(t) = P_0 \cdot r^t$

(b) Determine the population after **7 hours**.

16. (Spicy) A certain substance decays exponentially. Initially, there are **500 g** of the substance. After **6 hours**, **200 g** remain.

(a) Determine an exponential model of the form

$$A(t) = A_0 \cdot r^t$$

(b) How long will it take for the amount to decrease to **50 g**?

(c) Determine the **percent decrease per hour**.

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17. (Medium) Solve the equation:

$$2^{x+2} = 4 \cdot 3^x$$

18. (Spicy) Solve the equation:

$$2^{2x-1} = 3^x \cdot 2^x$$

Give your answer as an **exact value in logarithmic form**.

Exponents & Logarithms Review (Part 2)

19. (Medium) Solve the equation:

$$4 \cdot 3^x = 162$$

20. (Spicy) Solve the equation:

$$5^{2x-1} = 3^x \cdot 7$$

Give your answer as an **exact value in logarithmic form**.

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21. (Medium) Simplify the following expression:

$$\ln(4x^2) + \ln(3x)$$

Write your answer as a **single logarithm**.

22. (Spicy) Simplify the following expression completely:

$$\ln(2x^3) + 2\ln(3x) - \ln(12x^2) + \frac{1}{2}\ln(x)$$

Write your answer as a **single logarithm**.

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23. (Medium) Solve the equation:

$$\ln(x) + \ln(x-1) = \ln(6)$$

24. (Spicy) Solve the equation:

$$\ln(x+4) + \ln(x-2) = \ln(20)$$

Give your answer as an **exact value**.

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25. (Medium) A population grows continuously according to the model

$$N(t) = 500e^{0.2t}$$

(a) Determine the population after **6 hours**.

26. (Spicy) A population grows continuously according to the model

$$N(t) = 800e^{0.25t}$$

(a) How long will it take for the population to reach **10,000**?

(b) Determine the time required for the population to **triple**.

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27. (Medium) A radioactive substance decays according to

$$A(t) = A_0e^{-kt}$$

A sample starts with **100 g** and decreases to **70 g** after **4 hours**. Determine the value of k.

28. (Spicy) A radioactive substance decays according to $A(t) = A_0e^{-kt}$ A sample starts with **200 g**. After **5 hours**, **120 g** remain.

(a) Determine the value of k(exact value).

(b) How long will it take for the sample to decrease to **50 g**?

29. (Medium)
Determine whether the series is **convergent or divergent**:

$$5 + 2.5 + 1.25 + 0.625 + \cdots$$

If convergent, find its **sum**.

30. (Spicy) Consider the infinite geometric series: $6 + 3 + \frac{3}{2} + \frac{3}{4} + \cdots$

(a) Determine whether the series is **convergent or divergent**.

(b) If convergent, determine its **sum**.

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31. (Medium) A geometric series has: $a = 3, r = 2$

Find the sum of the first **6 terms**, S_6.

32. (Spicy) A geometric series has: $S_3 = 21$ and $S_4 = 45$

(a) Determine the common ratio r.

(b) Determine the first term a.

(c) Find S_8.

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33. (Medium) A student deposits **$1000 at the beginning of each year** into an account earning **5% annually**.

Determine the value after **8 years**.

34. (Spicy) A student deposits **$2000 at the beginning of each year** into an account earning **6% annually**.

(a) Determine the value after **12 years**.

(b) How many years will it take to exceed **$40,000**?
