Exponents & Logarithms Review Solutions (Part 1)

1. $f(x) = 2 \cdot 3^{x-1} - 4$

(a) y-intercept

Set $x = 0$:

$$f(0) = 2 \cdot 3^{-1} - 4 = 2 \cdot \frac{1}{3} - 4 = \frac{2}{3} - 4 = -\frac{10}{3}$$

Answer: $\left(0, -\frac{10}{3}\right)$

(b) End behavior

- $x \to \infty$: $3^{x-1} \to \infty \Rightarrow f(x) \to \infty$
- $x \to -\infty$: $3^{x-1} \to 0 \Rightarrow f(x) \to -4$

---

2. $g(x) = -4 \cdot 2^{-(x+2)} + 6$

(a) y-intercept

$$g(0) = -4 \cdot 2^{-2} + 6 = -4 \cdot \frac{1}{4} + 6 = -1 + 6 = 5$$

(b) End behavior

- $x \to \infty$: $2^{-(x+2)} \to 0 \Rightarrow g(x) \to 6$
- $x \to -\infty$: $2^{-(x+2)} \to \infty \Rightarrow g(x) \to -\infty$

(c) Growth or decay

Base effectively 2^{-x} ⇒**decay**

---

3. Fish model

$$P(t) = 1200r^t$$

Given:

$$3000 = 1200r^5 \Rightarrow r^5 = 2.5 \Rightarrow r = 2.5^{1/5}$$

(b)

$$P(10) = 1200(2.5^2) = 1200(6.25) = 7500$$

---

4. Investment

$$13200 = 5000r^8 \Rightarrow r^8 = 2.64 \Rightarrow r = 2.64^{1/8}$$

(b)

$$25000 = 5000r^t \Rightarrow 5 = r^t \Rightarrow t = \frac{\log 5}{\log r}$$

(c)

$$\text{rate} = r - 1$$

---

5. Decibels

$$80 - 60 = 20 = 10\log\left(\frac{I_B}{I_A}\right)$$

$$2 = \log\left(\frac{I_B}{I_A}\right) \Rightarrow \frac{I_B}{I_A} = 100$$

6.

(a)

$$dB = 72 + 10\log(250) \approx 72 + 23.98 = 95.98$$

(b)

$$92 - 72 = 20 = 10\log\left(\frac{I}{I_A}\right) \Rightarrow \frac{I}{I_A} = 100$$

7.

$$\log_2(5x) + \log_2(3x^2) - \log_2(4) = \log_2\left(\frac{15x^3}{4}\right)$$

8.

$$3\log_5(2x^2) = \log_5(8x^6)$$

$$\frac{1}{2}\log_5(25x^3) = \log_5(5x^{3/2})$$

Combine:

$$\log_5\left(\frac{8x^6 \cdot 5x^{3/2}}{5x \cdot 16}\right) = \log_5\left(\frac{8x^{7.5}}{16}\right) = \log_5\left(\frac{x^{7.5}}{2}\right)$$

9.

$$\log_3\big(x(x-2)\big) = 2$$
$$\Rightarrow x(x-2) = 9$$
$$x^2 - 2x - 9 = 0$$
$$\Rightarrow x = 1 \pm \sqrt{10}$$
Valid: $x > 2$
$$\Rightarrow x = 1 + \sqrt{10}$$

10.

$$\log_2\big((x+3)(x-1)\big) = 3$$
$$\Rightarrow (x+3)(x-1) = 8$$
$$x^2 + 2x - 3 = 8$$
$$\Rightarrow x^2 + 2x - 11 = 0$$
$$x = -1 \pm \sqrt{12}$$
Valid: $x = -1 + \sqrt{12}$

11.

$$\log_a(b^2c) = 2m + n$$

12.

$$\log_a\left(\frac{b\sqrt[3]{c}}{a^2c^4}\right) = m + \frac{1}{3}n - 2 - 4n = m - \frac{11}{3}n - 2$$

13.

$$5 \cdot 2^x = 80 \Rightarrow 2^x = 16 \Rightarrow x = 4$$

14.

$$3^{2x-1} = 5^{x+2}$$

Take logs:

$$(2x-1)\log\ 3 = (x+2)\log\ 5$$

Solve:

$$2x\log\ 3 - x\log\ 5 = \log\ 3 + 2\log\ 5$$
$$x = \frac{\log\ 3 + 2\log\ 5}{2\log\ 3 - \log\ 5}$$

15.

$$2700 = 900r^4 \Rightarrow r^4 = 3 \Rightarrow r = 3^{1/4}$$
$$P(7) = 900 \cdot 3^{7/4}$$

16.

$$200 = 500r^6 \Rightarrow r^6 = 0.4 \Rightarrow r = 0.4^{1/6}$$

(b)

$$50 = 500r^t \Rightarrow 0.1 = r^t \Rightarrow t = \frac{\log 0.1}{\log r}$$

(c)

$$\text{decay rate} = 1 - r$$

17.

$$2^{x+2} = 4 \cdot 3^x \Rightarrow 2^x \cdot 4 = 4 \cdot 3^x$$
$$2^x = 3^x \Rightarrow x = 0$$

18.

$$2^{2x-1} = 3^x \cdot 2^x \Rightarrow 2^{x-1} = 3^x$$

Logs:

$$(x-1)\log\ 2 = x\log\ 3$$
$$x = \frac{\log 2}{\log\ 2 - \log\ 3}$$

Exponents & Logarithms Review (Part 2)

19.

$$4 \cdot 3^x = 162 \Rightarrow 3^x = 40.5 \Rightarrow x = \log_3(40.5)$$

20.

$$5^{2x-1} = 7 \cdot 3^x$$
$$(2x-1)\log\ 5 = x\log\ 3 + \log\ 7$$
$$x = \frac{\log\ 7 + \log\ 5}{2\log\ 5 - \log\ 3}$$

21.

$$\ln(4x^2) + \ln(3x) = \ln(12x^3)$$

22.

$$= \ln\left(\frac{2x^3(3x)^2\sqrt{x}}{12x^2}\right) = \ln(3x^{3.5})$$

23.

$$\ln(x(x-1)) = \ln\ 6 \Rightarrow x^2 - x - 6 = 0 \Rightarrow x = 3$$

24.

$$(x+4)(x-2) = 20 \Rightarrow x^2 + 2x - 28 = 0 \Rightarrow x = -1 + \sqrt{29}$$

25.

$$N(6) = 500e^{1.2}$$

26.

$$10000 = 800e^{0.25t} \Rightarrow t = \frac{\ln(12.5)}{0.25}$$

Tripling:

$$3 = e^{0.25t} \Rightarrow t = \frac{\ln 3}{0.25}$$

27.

$$70 = 100e^{-4k} \Rightarrow k = -\frac{\ln(0.7)}{4}$$

28.

$$120 = 200e^{-5k} \Rightarrow k = -\frac{\ln(0.6)}{5}$$

(b)

$$50 = 200e^{-kt} \Rightarrow t = \frac{\ln(0.25)}{-k}$$

29.

$$a = 5,\ r = 0.5 \Rightarrow S = \frac{5}{1 - 0.5} = 10$$

30.

$$a = 6,\ r = \frac{1}{2} \Rightarrow S = \frac{6}{1 - \frac{1}{2}} = 12$$

31.

$$S_6 = 3\frac{1 - 2^6}{1 - 2} = 3(63) = 189$$

32.

$$S_3 = a(1 + r + r^2) = 21$$
$$S_4 = S_3 + ar^3 = 45 \Rightarrow ar^3 = 24$$

Solve $\rightarrow r = 2,\ a = 3$

$$S_8 = 3\frac{1 - 2^8}{1 - 2} = 3(255) = 765$$

33.

Annuity due:

$$S = 1000(1.05)\frac{1.05^8 - 1}{0.05}$$

34.

$$S = 2000(1.06)\frac{1.06^{12} - 1}{0.06}$$

(b)

Solve:

$$40000 = 2000(1.06)\frac{1.06^n - 1}{0.06}$$

$\rightarrow$ solve numerically.