Exponential and Logarithmic Problems

1. Joe invested $4500 at a fixed annual interest rate compounded annually. At the end of 12 years, the investment has doubled in value. To the nearest hundredth of a percent, Joe's investment pays interest at _____% / year compounded annually.

2. The population of a particular town on July 1, 2011 was 20 000. If the population decreases at an average annual rate of 1.4%, how long, to the nearest whole year, will it take for the population to reach 15 300?

3. Sodium-24 decays from 148 g to 5.2 g in 72.5 hours. Algebraically, determine the half-life of Sodium-24, to the nearest hour.

4. When a radioactive element decays, the mass remaining, $m(t)$, is expressed by the formula $m(t) = c(0.5)^{\frac{t}{p}}$, where c is the initial mass, t is the time in hours and p is the half-life of the element. If a 100 mg sample of Radon-211 (Rn^{211}) decays to 30 mg after 29 hours, then find the half-life of Rn^{211}, correct to one decimal place.

5. A student attempts to solve the equation:

$$\log_6(x-5) + \log_6(x-6) = 1$$

Their work is shown below:

$$log_6(x-5)(x-6) = 1$$
$$6^1 = x^2 - 11x + 30$$
$$x^2 - 11x + 24 = 0$$
$$(x-8)(x-3) = 0$$
$$x = 8, x = 3$$

a) Identify the error in the student's solution.
b) Explain why one of the solutions must be rejected.
c) State the correct solution.

6. Solve each of the following and rank the solutions in order from least to greatest.

a) $\boldsymbol{log_x 64 = \frac{3}{2}}$

b) $\boldsymbol{log_{\frac{1}{k}}\left(k^{-\frac{1}{2}}\right) = x}$

c) $\boldsymbol{7^{log_7 8} = x}$

d) $\boldsymbol{\frac{2^{log\,1000}}{2^{log\,10}} = x}$

_____ _____ _____ _____

7. Solve algebraically $log_7(x+1) + log_7(x-5) = 1$.

8. $log_5(2x+1) - log_5(x-2) = 1$. Algebraically determine the exact value of x.

9. $\frac{1}{3}log_2 27 + log_2 k - 2\,log_2 m - log_2 12$ Rewrite as a single logarithmic function.

10. Using laws of logarithms, the expression $log_4 15 + 2\,log_4 9 - log_4 243$ can be written as a single logarithm in the form of $log_4 m$. To the nearest whole number, the value of m is _____.

11. Solve the following equation algebraically. $4^{4x} = 8^{x+5}$

12. Algebraically solve the equation $8^{3x+4} = 4^{x-9}$. Express your answer as an exact value.

13. An investment of $1000 grows at a rate of 9.5% per year. Algebraically determine how many years will it take for the investment to be worth $50 000. Round your answer to the nearest tenth of a year.

14. Sound loudness is measured in a special scale called the decibel scale. The decibel reading, dB, is determined by the formula $dB = 10\,log(I)$, where I is the intensity of the sound. If a chainsaw has a decibel reading of 100 dB and heavy traffic has a decibel reading of 85 dB, then how many times more intense is the chainsaw than heavy traffic?

15. The pH of a solution can be determined using the formula $pH = -log_{10}[H^+]$, where $[H^+]$ is the concentration of hydrogen ions in the solution in mol/L. The pH of normal rainwater is considered to be 5.6.
    a) What is the value of the concentration of hydrogen ions in normal rainwater?
    b) What is the pH of a solution where the concentration of hydrogen ions is double that of normal rainwater?

16. An SUV is purchased for $65 000. Suppose the annual rate of depreciation of the vehicle is 14%.
    a) Write a function equation in the form $y = ab^x$ that could be used to model the actual value of the SUV for years after the purchase.
    b) Use your function equation to determine the number of years after the purchase it will take for the SUV to only be worth $25 000.

17. If the domain of $f(x)$ is $x \geq -5$ and the domain of $g(x)$ is $x \geq 7$, then the domain of $h(x) = f(x) - g(x)$ is
    a. $x \in R$
    b. $x \geq 7$
    c. $x \geq -5$
    d. $-5 \leq x \leq 7$

18. The following statements were made about the function $y = log_a x\,, where\ a > 1$.
    1. The function has a y-intercept of 1.
    2. The function increases in value as x increases.
    3. The function has an asymptote of $x = 0$.
    4. The range of the function is $\{y \mid y > 0, y \in R\}$.
    5. The function contains the point $(a, 1)$.

    The statements that are true are numbered _____, _____, and _____.