Infinite Series

Part A — Understanding Infinite Series

1. When does a geometric series have a finite sum? State the condition on r.

Part B — Determining Convergence

2. Determine whether each series is convergent or divergent. Justify your answer:

$$50 + 45 + \frac{81}{2} + \frac{729}{20} + \cdots$$

3. For what values of $x$will the series have a finite sum?

$$1 + \frac{x-2}{3} + \left(\frac{x-2}{3}\right)^2 + \left(\frac{x-2}{3}\right)^3 + \cdots$$

Part C — Direct Infinite Sums (Core Skills)

4. Find the sum of each infinite geometric series. Show all work:

a) $27 + 9 + 3 + 1 + \cdots$

b) $1 - 0.5 + 0.25 - \frac{1}{8} + \cdots$

c) $\frac{14}{3} + \frac{7}{3} + \frac{7}{6} + \cdots$

d) $\frac{125}{64} - \frac{25}{16} + \frac{5}{4} - \cdots$

Part D — Fraction-Based Series

5. Find the sum:

$$12 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \cdots$$

Part E — Reverse Problems

6. The sum of an infinite geometric series is 1 and the common ratio is $-\frac{2}{5}$.
   Find the first three terms.

7. An infinite geometric series has $S_\infty = 24$and $r = \frac{3}{4}$.
   Find the 8th term.

Part F — Algebra with Infinite Series

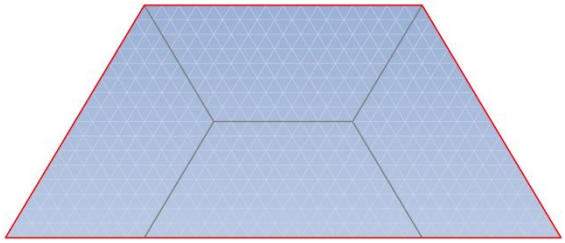
8. Given $a = 12$and $S_\infty = 350$, find r.

9. If $1 + x + x^2 + x^3 + \cdots = 20$, find x.

10. Given $S_3 = 19$and $S_\infty = 27$, find r.

Part G — Visual / Pattern Models

11. A trapezoid is repeatedly divided into four equal parts, and one part is shaded each time. Write an equation for $S_\infty$and determine the total shaded area.



Part H — Repeating Decimals (Geometric Series)

12. Convert each repeating decimal into a fraction using Newton's Method:
    - Write the number as an expanded number with lots of decimal places.
    - Write this number as the sum of infinite decimals.
    - Write this number as the sum of infinite fractions.
    - Identify the a and r for the series.
    - Use the infinite geometric series formula and simplify it to find the fraction.

    a) $0.0\overline{3}$ b) $5.\overline{36}$ c) $0.\overline{87}$

Part I — Applications

13. A ball is dropped from a height of 2.0m. After each bounce, it rises to 63% of its previous height.
    a) Total distance after 5 bounces
    b) Total distance until it comes to rest

14. An oil well produces 30,000 barrels in its first month and decreases by 5% each month.
    a) Total production before it runs dry
    b) When production drops below 100 barrels

15. A worker can be paid either:
    - $5000 per day for 100 days
    - or $0.01, $0.02, $0.04, … for 100 days
      a) Which is better?