

Geometric Series

1. Derive the formula for the sum of a geometric series.
2. Explain each variable in $S_n = \frac{a(1-r^n)}{1-r}$
3. What does $S_{n+1} - S_n$ represent?

Section 1 — Using the Sum Formula: *Apply the geometric sum formula.*

4. Find each sum:
 - a) $1 + 2 + 4 + 8 + 16 + 32 + 64 + 128$
 - b) $1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{128}$
 - c) $2 + 6 + 18 + 54 + 162 + 486 + 1458$
5. Find:
 - a) S_6 for $a = 4, r = 3$
 - b) S_{20} for $18 + 9 + 4.5 + \dots$
 - c) S_{10} for $5 + 10 + 20 + 40 + \dots$

Section 2 — Algebra Foundations: *Solve for r , a , or n .*

6. Given:
 - a) $a = 12, r = 2, S_n = 762$, find n
 - b) $r = -2, S_8 = 1020$, find a

Section 3 — Algebra with Series (Multi-Step): *Combine multiple ideas.*

7. Given $S_2 = 5$ and $S_4 = 85$, find r
8. Given: $t_2 = 4, t_4 = 36$. Find a , Find r , Find S_{10}

Section 4 — Non-Standard Series (Radicals): *Work with non-integer ratios.*

9. Solve: $\sqrt{2} + 2 + 2\sqrt{2} + \dots + 256$
10. Find the sum of $250 + 50 + 10 + \dots + \frac{2}{625}$
11. Find:
 - a) S_6 for $a = 3, r = -2$
 - b) $-\frac{1}{8} + 0.25 - 0.5 + \dots + 256$

Applications of Geometric Series

Section 1 — Applications (Single-Step Models): *Translate context into a geometric model.*

12. A ball is dropped from 2.0 m. Each bounce reaches 63% of the previous height. Find the height after 5 bounces.
13. A fractal starts at 972 cm and each stage is $\frac{1}{3}$ of the previous. Find the length at stage 9.
14. A filtering process removes 18% each cycle. Starting with 700 g, how much remains after 5 cycles?
15. An oil well produces 30,000,000 barrels in the first month and decreases by 5% each month. Find the total production after 20 years.

Section 2 — Applications: *Interpret and compare series.*

16. A contest offers:
 - \$20,000,000 immediately
 - OR \$1, \$2, \$4, \$8, ... for 30 yearsAfter how many years does the second option exceed the first?
17. How would you find the sum of $2 + 13 + 17 + 4 + 8 + 16 + \dots + 1024$?
18. An oil well produces 36,000 barrels in the first month and 95% each month after. Find total production in one year.
19. A ball is dropped from 2.0 m and rebounds to 60% each time. Find the total distance travelled after 5 bounces.
20. A ball is dropped from 3 m and rebounds to 85% each time. Find the total distance travelled after 7 bounces.
21. A tree grows 1.2 m in the first year and 75% each year after. Find the total height when the last growth is 0.02138154 m.