Logarithms Laws

- Allow the simplification of logarithmic expressions

① $\log_a a = 1 \rightarrow a^1 = a$

② $\log a^m = m \cdot \log a$

$\log_2 2^3 = 3 \cdot \log_2 2$

③ $\log_{25} 125 = \frac{\log_5 125}{\log_5 25}$

$\log_a b = \frac{\log_c b}{\log_c a}$

1 Example: $\log_2 8$ → $\log_2 2^3$ → $3 \cdot \log_2 2 = 1$, $3 \cdot 1 = 3$

8 Example: $\log_5 \left(\frac{1}{25}\right)$ → $\log_5 \left(\frac{1}{5^2}\right)$ → $\log_5 (5)^{-2}$ → $-2 \cdot \log_5 (5) = 1$, $-2 \cdot 1 = -2$

12 example: $\log_7 (\sqrt{343})$ → $\log_7 (343^{\frac{1}{2}})$ → $\frac{1}{2} \log_7 343$ → $\frac{1}{2} \log_7 (7)^3$ → $3 \cdot \frac{1}{2} \log_7 7 = 1$, $3 \cdot \frac{1}{2} \cdot 1 = 1.5$

15 example: $\log_2 (16\sqrt{2})$ → $\log_2 (2^4 \cdot 2^{\frac{1}{2}})$ → $\log_2 (2^{4.5})$ → $4.5 \cdot \log_2 2 = 1$, $4.5 \cdot 1 = 4.5$

example 37: $\log_2 \sqrt{32}$ → $\log_2 (32)^{\frac{1}{2}}$ → $\frac{1}{2} \log_2 (2)^5$ → $5 \cdot \frac{1}{2} \log_2 2 = 1$, $5 \cdot \frac{1}{2} \cdot 1 = 5/2$ (2.5)

example 67: $\log_2 \left(\frac{\sqrt{A} \cdot B^3}{\sqrt[3]{C} \cdot D^2}\right)$

$\log_2 (\sqrt{A} \cdot B^3) - \log_2 (\sqrt[3]{C} \cdot D^3)$

$\log_2 (A^{\frac{1}{2}}) + \log_2 (B^3) - (\log_2 C^{\frac{1}{3}} + \log_2 (D^3))$

$\frac{1}{2} \log_2 A + 3 \log_2 B - \frac{1}{3} \log_2 C - 2 \log D$