Foundations of Logarithms

$$\log_a a = 1 \qquad \log_a b = \frac{\log_c b}{\log_c a} \qquad \log a^m = m \log a$$

Evaluate each expression **without a calculator**.

1. $\log_2(8)$
2. $\log_5(125)$
3. $\log_3(27)$
4. $\log_2(32)$
5. $\log_{10}(10000)$
6. $\log_4(64)$
7. $\log_2\left(\frac{1}{8}\right)$
8. $\log_3\left(\frac{1}{27}\right)$
9. $\log_5\left(\frac{1}{25}\right)$
10. $\log_5(\sqrt{25})$
11. $\log_4(\sqrt{64})$
12. $\log_7(\sqrt{343})$
13. $\log_4(8)$
14. $\log_9(27)$
15. $\log_2(16\sqrt{2})$

Logarithm Laws

$$\log(ab) = \log a + \log b \qquad \log\left(\frac{a}{b}\right) = \log a - \log b$$

Apply logarithm laws to simplify expressions Simplify each expression **as much as possible**. Do **not** approximate.

16. $\log_3(4) + \log_3(5)$
17. $\log_2(3) + \log_2(8)$
18. $\log_5(2) + \log_5(25)$
19. $\log_3(12) - \log_3(4)$
20. $\log_5(20) - \log_5(5)$
21. $\log_2(18) - \log_2(9)$
22. $\log_5\left(\frac{125}{5}\right)$
23. $\log_2\left(\frac{2^6}{2^3}\right)$
24. $2\log_2(3)$
25. $3\log_5(2)$
26. $\frac{1}{2}\log_3(16)$
27. $\log_3(8) + 2\log_3(3)$
28. $\log_2(12) - \log_2(3) + \log_2(2)$
29. $2\log_5(10) - \log_5(4)$
30. $\log_3(27) + \log_3(9)$
31. $\log_2(16) - \log_2(2) + \log_2(8)$
32. $\log_5(100) - \log_5(4) + \log_5(5)$

Simplify by recognizing structure before applying log laws. Simplify each expression **as much as possible**. Do **not** approximate.

33. $\log_3(3^5)$
34. $\log_4(4^{-2})$
35. $\log_5(5^{1/2})$
36. $\log_3(\sqrt{27})$
37. $\log_2(\sqrt{32})$
38. $\log_4(8)$
39. $\log_3(27 \cdot 9)$
40. $\log_2(16\sqrt{2})$
41. $\log_3(3^2 \cdot 3^5)$
42. $\log_4((4^3)^2)$
43. $\log_x(x^4)$
44. $\log_a(a^3b^2)$
45. $\log_{x+1}((x+1)^3)$
46. $log_8\left(\frac{1}{4}\right)$
47. $-log_2\left(\frac{125}{8}\right)$

Apply log laws algebraically. Simplify each expression **as much as possible**. Do **not** approximate.

48. $\log(a) + \log(b)$
49. $\log(x) - \log(y)$
50. $2\log(m)$
51. $\log(a) + \log(b) - \log(c)$
52. $2\log(x) + \log(y)$
53. $\log(p) + 2\log(q) - 3\log(r)$
54. $\frac{1}{2}\log(a)$
55. $\frac{3}{2}\log(x)$
56. $\frac{1}{2}\log(a) + 2\log(b)$
57. $\log(x^3) + \log(x^2)$
58. $\log(x^6) - \log(x^3)$
59. $2\log(a) - 3\log(b)$
60. $log_2\left(16\sqrt{128}\right) + log_7\left(49\sqrt[3]{7}\right)$
61. $log_3 12 - log_3 4 - log 10$
62. $\log_2(8x)$
63. $-32\, log_2\left(\frac{1}{8}\right)$
64. $\log_2(10) - \log_4(12) + \log_5(8)$
65. $\frac{1}{2} log_3 a + 2\, log_3 b - 3\, log_3 c - \frac{1}{3} log_3 d$
66. $\log_2\left(\frac{AB}{C}\right)$
67. $log_2\left(\frac{\sqrt{A}B^3}{\sqrt[3]{C}D^2}\right)$
68. $log_3\sqrt{27 \cdot 4} - 0.5\, log_3 4 + log_9 81$