Applications of Exponential Equations

Population → $P_f = P_i r^{periods}$

1. A population of 20 rabbits is released into a wildlife region. The population triples each year for 5 years. What is the population after 5 years?

2. The growth of a new social networking site can be modelled by the exponential function $N(t) = 1.1^t$, where N is the number of users after t days. How long will it take, to the nearest day, for the number of users to exceed 1 000 000?

3. The population of a city is increasing at a constant rate of 5% per year. The city's present population is 200 000. Determine the minimum number of years it will take for the population to exceed 500 000.

4. According to a Statistics Canada report released in 2010, Saskatoon had the fastest-growing population in Canada, with an annual growth rate of 2.77%. At this rate, approximately how long would it take for Saskatoon's population to grow by 25%?

5. Statistics indicate that the world population since 1995 has been growing at a rate of about 1.27% per year. United Nations records estimate that the world population in 2011 was approximately 7 billion. Assuming the same exponential growth rate, when will the population of the world be 9 billion?

6. A bacteria culture starts with 5000 bacteria. After 6 hours, the estimated count is 80,000. What is the doubling period for this bacteria culture?

7. On Monday morning, Julia found that a colony of bacteria covered an area of 100 cm^2 on the agar. After 10 h, she found that the area had increased to 200 cm^2. Assume that the growth is exponential.
   - By Tuesday morning (24 h later), what area do the bacteria cover?
   - Consider Earth to be a sphere with radius 6378 km. How long would these bacteria take to cover the surface of Earth?

Investment → $A = P(r)^{periods}$

8. A $1000 investment earns interest at a rate of 8% per year, compounded quarterly.
   a) Write an equation for the value of the investment as a function of time, in years.
   b) Determine the value of the investment after 4 years.

9. A $1000 investment earns interest at a rate of 8% per year, compounded quarterly.
   - Write an equation for the value of the investment as a function of time, in years.
   - Determine the value of the investment after 4 years.
   - How long will it take for the investment to double in value?

10. Explain why more income is generated when interest is compounded semi-annually vs monthly vs daily vs continuously.

11. Simionie needs $7000 to buy a snowmobile, but only has $6000. His bank offers a GIC that pays an annual interest rate of 5%, compounded annually. How long would Simionie have to invest his money in the GIC to have enough money to buy the snowmobile?

12. A computer, originally purchased for $3000, depreciates in value according to the function $V(t) = 3000\left(\frac{1}{2}\right)^{\frac{t}{3}}$, where V is the value, in dollars, of the computer at any time, t, in years. Approximately how long will it take for the computer to be worth 10% of its purchase price?

13. Suzie plans to grow her investment from $50,000 to $85,000 in 2 years. An investment portfolio compounds their interest monthly. How much does the annual interest rate need to be for Suzie to meet her investment goals?

14. BMO offers a mutual fund that gives an annual interest rate of 5.75% compounded semi-annually. At a local credit union, they offer investments that are compounded daily. At what interest rate should the credit union offer if the two institutions are to offer the same investment returns?

Half Lives → $m_f = m_i 0.5^{\frac{t}{half-life}}$

15. Some household smoke detectors contain a small amount of the radioactive element Americium 241. Am-241 has a half-life of approximately 432 years. The average smoke detector contains 200µg of Am-241. Write an exponential function that models the decay of Am-241.

16. Nickel-65 (Ni-65) has a half-life of 2.5 h.
    - Write an exponential function to model this situation.
    - What fraction of a sample of Ni-65 will remain after 10 h?

17. Francium 87, is a radio active element that has a half life of 22 minutes. It decays into radium 223 that is used for radiation treatments to kill cancer cells. Francium appears naturally in the earth's crust in Uranium mines. Suppose you found 55g of francium, how long will it take to decay to 1g?

18. C-14 has a half life of 5370 years. C12 is stable and does not decay. Living fossils begin with equal amounts of C12 and C14. A large piece of fossil was found to contain 1200kg of C12 and 500g of C14. How old is this fossil?

19. A radio active substance decayed from 600g to 580g in 15 years.
    a) What is the half-life of the radio active substance?
    b) How much of this radio active substance will be left after 100 years?
    c) After how many years will there be only 1% of the substance left?