Natural Logarithms

Many real-world situations in this assignment follow exponential models of the form:

Decay (decreasing quantity)

$$A(t) = A_0 e^{-kt}$$

Growth (increasing quantity)

$$N(t) = N_0 e^{rt}$$

Level 1 — Solving for Time (Decay Models)

1. A capacitor follows $V(t) = 10e^{-t/2}$. How long until the voltage is 1?

2. A chemical reaction follows $A(t) = 0.50e^{-0.04t}$. How long until the concentration is 0.10?

3. Light intensity follows $I(x) = e^{-0.8x}$. How thick must the material be to reduce the intensity to 0.20?

Level 2 — Solving for Parameters

4. A capacitor follows $V(t) = V_0 e^{-t/RC}$. If $V_0 = 12$and the voltage drops to 3 in 4 units of time, find RC.

5. Light intensity follows $I(x) = I_0 e^{-\alpha x}$. If 70% of the light is transmitted through 1 cm, find the absorption coefficient α.

6. A chemical reaction follows $A(t) = A_0 e^{-kt}$. If the concentration drops from 2.0 to 0.5 in 30 seconds, find k.

Level 3 — Growth Models

7. A population triples in 5 hours. Find the growth rate r.

8. A bacterial culture follows $N(t) = 200e^{0.3t}$. How many hours will it take to reach 10,000 cells?

9. A bacterial culture grows from 10,000 to 80,000 cells in 6 hours. Find the growth rate r.

Level 4 — Logarithmic Models (pH)

10. A solution has pH 3 and another has pH 6. How many times more acidic is the first solution?

11. The pH of a solution is given by $pH = -\frac{\ln [H^+]}{\ln 10}$. Find the pH when $[H^+] = 4 \times 10^{-6}$.

12. A solution has pH = 5.2.. Find the concentration $[H^+]$.

Level 5 — Multi-Step Exponential Solving

13. A reliability model is given by $R(t) = e^{-t/2000}$. How many hours until $R(t) = 0.70$?

14. A substance follows $A(t) = A_0 e^{-0.2t}$. How long until 15% of the original amount remains?

15. A tank follows $C(t) = 30e^{-(10/500)t}$. How long until the concentration reaches 5?

Level 6 — Parameter + Model Substitution

Helpful Information

For exponential decay: $N(t) = N_0 e^{-\lambda t}$

- $\lambda$is called the **decay constant** — it controls how fast something decays.
- A **half-life** is the time it takes for a quantity to be reduced to half its original value.

If a substance has half-life T, then: $\frac{1}{2} = e^{-\lambda T}$. You can use this equation to find λ.

16. A substance has a half-life of 10 years. Find the decay constant λ.

17. Using this same substance, how long will it take for 25% to remain?

18. Carbon-14 decay follows $N(t) = N_0 e^{-\lambda t}, \lambda = \frac{\ln 2}{5730}$. If 40% of the original sample remains, how old is the sample?

Level 7 — Full Structure (Capstone)

19. A medication follows $M(t) = 30e^{-0.8t}$. How much remains after 3 hours?

20. Using the same model, how long until only 1 mg remains?

21. A tank model is given by $C(t) = C_{in} + (C_0 - C_{in})e^{-(Q/V)t}$. A 200 L tank starts with pure water. Brine with concentration 80 mg/L flows in at 4 L/min.
How long until the concentration reaches 40 mg/L?