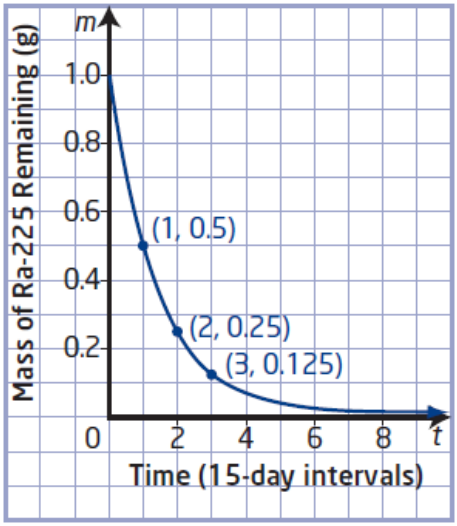
Exponential & Logarithmic Word Problems

1. A flu virus is spreading through the student population of a school according to the function $N = 1(2)^t$, where N is the number of people infected and t is the time, in days.
   - How many people have the virus after 1 day? after 4 days? after 10 days?
   - Sketch a of graph the function.
   - When will 500 students be infected?

2. The Brandt's are a young couple just getting started in planning their financial future. They have decided to take two steps towards their goal. They have $1000 to invest in a compound interest investment. They found a fund that promises them 20% per annum for the duration of their investment.
   - Sketch a graph of their investment over ten years.
   - Determine the equation in the form $P(x) = ar^x$, where x is the time in years.

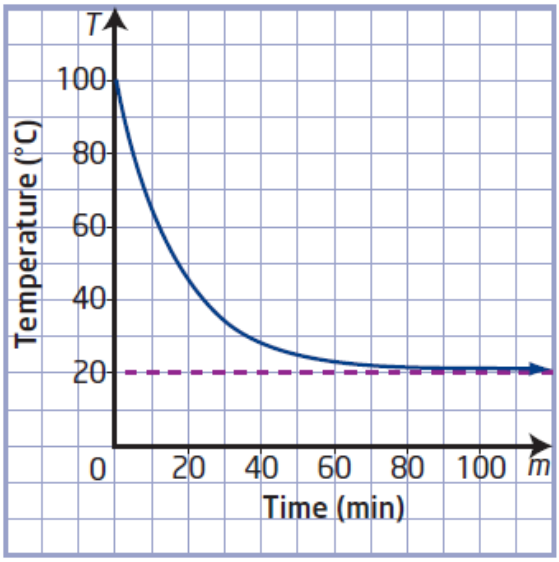
3. The measure of the acidity of a solution is called its pH. The pH of swimming pools needs to be checked regularly. This is done by measuring the concentration of hydrogen ions [H+] in the water. The relationship between the hydrogen ion concentration, H, in moles per litre (mol/L), is
   $$\log_{0.1}[\mathrm{H}+] = \mathrm{pH}$$
   - Sketch the graph of this function.
   - Water with a pH of less than 7.0 is acidic. What is the hydrogen ion concentration for a pH of 7.0?
   - Water in a swimming pool should have a pH of between 7.0 and 7.6. What is the equivalent range of hydrogen ion concentration?

4. A radioactive sample of radium (Ra-225) has a half-life of 15 days. The mass, m, in grams, of Ra-225 remaining over time, t, in 15-day intervals, can be modelled using the exponential graph shown.
   a) What is the initial mass of Ra-225 in the sample?
   b) What value does the mass of Ra-225 remaining approach as time passes?
   c) Write the exponential decay model $m(x) = ar^x$ that relates the mass of Ra-225 remaining to time, in 15-day intervals.
   d) Estimate how many days it would take for Ra-225 to decay to $\frac{1}{30}$ of its original mass.



5. According to a Statistics Canada report released in 2010, Saskatoon had the fastest-growing population in Canada, with an annual growth rate of 2.77%.
   - What function $P(x) = ab^x$ could be used to model this situation?
   - Sketch a graph of the function.
   - At this rate, approximately how long would it take for Saskatoon's population to grow by 25%?

6. A cup of water is heated to 100 °C and then allowed to cool in a room with an air temperature of 20 °C. The temperature, T, in degrees Celsius, is measured every minute as a function of time, m, in minutes, and these points are plotted on a coordinate grid. It is found that the temperature of the water retains only 75% of its temperature difference every 5 minutes. A smooth curve is drawn through the points, resulting in the graph shown.



   a) What is the transformed exponential function in the form $y = a(r)^x + q$ that can be used to represent this situation?
   b) Describe how each of the parameters in the transformed function relates to the information provided.

7. The Brandt's also need a reliable vehicle; the loan officer has given them the formula $C(x) = 19000\left(\frac{8}{9}\right)^x$ where x is the time in years, to define the term of their loan.
   - Sketch a graph of the loan for their car over ten years.
   - Determine the point in which the Brandt's car is worth $5000.

8. A $100,000 used vehicle depreciates at a rate of 10% each year.

   a) Write an equation for the value of the car as a function of "t" years after purchase.
   b) What is the value of the car after 5 years due to depreciation?
   c) Solve using logarithms: How long will it take the vehicle to be worth 50% of its purchase price?
   d) Graph the value of the vehicle as a function time "t' years

9. If seafood is not kept frozen (below 0°C), it will spoil due to bacterial growth. The relative rate of spoilage increases with temperature according to the model $R = 100(2.7)^{T/8}$, where T is temperature (°C)and $R$is the relative spoilage rate:

   a) Sketch the graph of Rversus T for $0 \leq T \leq 25$. Clearly label key points.
   b) At what temperature will the relative spoilage rate double to 200?
   c) Determine the relative spoilage rate at $\mathbf{15°C}$.
   d) The maximum acceptable spoilage rate is $\mathbf{500}$. Explain what this temperature means in context. Should seafood ever be stored near this temperature?

10. Bonnie invested $250,000 in a stock that pays 2.25% in dividends quarterly (every 3 months). The dividends are invested back into the stock. Suppose the stock price did not grow much in value.

   a) What is the rate of growth of her investment?
   b) How much will she have earned from dividends after 5 years?
   c) Solve using logarithms: How long will it take for her investment to double in value?
   d) Solve using logarithms: How long will it take for her investment to increase to $1 million dollars?