Formula Sheet – Pre-Calculus 12

Linear Functions

Calculating Slope

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept Form

$$y = mx + b$$

Sketching Method:

- Identify the slope m and the y-intercept b.
- Plot the y-intercept at $(0, b)$.
- Use the slope $m = \frac{rise}{run}$ to plot a second point.
- Draw the line through the points.

Slope-Point Form

$$y - y_1 = m(x - x_1)$$

Sketching Method:

- Identify the point (x_1, y_1) and the slope m.
- Plot the point (x_1, y_1) on the graph.
- Use the slope $m = \frac{rise}{run}$ to plot a second point.
- Draw the line through the points.

General Form

$$Ax + By = C$$

Sketching Method:

- Find the y-intercept: set $x = 0$ and calculate y.
- Find the x-intercept: set $y = 0$ and solve for x.
- Plot both intercepts.
- Draw the line connecting them.

Quadratic Functions

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant: $b^2 - 4ac$

General Form

$$y = ax^2 + bx + c$$

Sketching Method

- Find the **axis of symmetry** using $x = -\frac{b}{2a}$
- Find the **vertex** with $f(\text{axis of symmetry})$.
- Find the **y-intercept** with $f(0)$.

OR

Sketching Method 2

- Identify a, b, and c.
- Identify the **x-intercepts:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Find the **axis of symmetry** by averaging the x-intercepts.
- Find the **vertex** with $f(\text{axis of symmetry})$.

Standard (Vertex) Form

$$y = a(x - p)^2 + q$$

Sketching Method 1

- Read the vertex (h, k).
- Draw the axis of symmetry, vertically through the vertex.
- Find the **y-intercept** with $f(0)$.

Factored Form

$$y = a(x - r_1)(x - r_2)$$

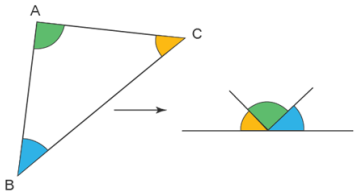
Sketching Method

- Identify the **x-intercepts** by solving for x in each of the binomials.
- Find the **axis of symmetry** by averaging the x-intercepts.
- Find the **vertex** with $f(\text{axis of symmetry})$.

Common Angle Relationships

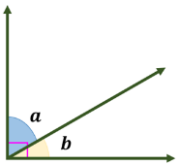
Triangle Angle Sum

$$\angle A + \angle B + \angle C = 180°$$



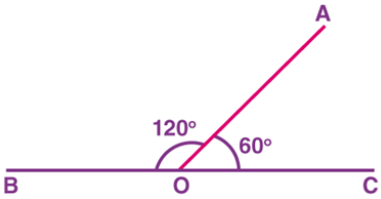
Complementary Angles

$$A + B = 90°$$



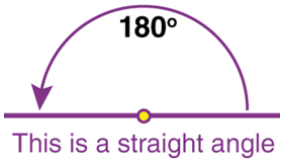
Supplementary Angles

$$A + B = 180°$$



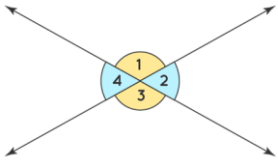
Straight Angles

Straight angle $= 180°$



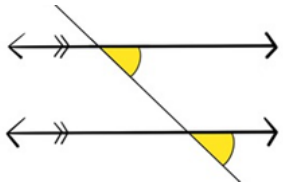
Vertically Opposite Angles

$$\angle A = \angle B$$



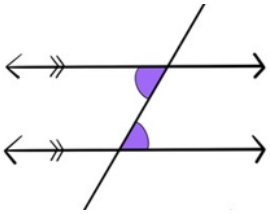
Corresponding Angles

$$\angle A = \angle B$$



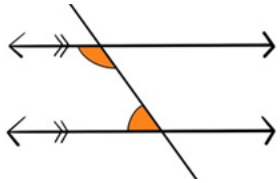
Alternate Interior Angles

$$\angle A = \angle B$$



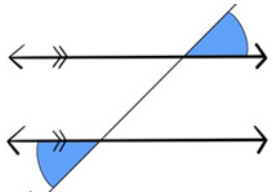
Co-Interior Angles

$$\angle A + \angle B = 180°$$



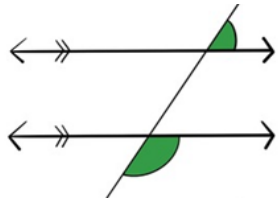
Alternate Exterior Angles

$$\angle A = \angle B$$



Co-Exterior Angles

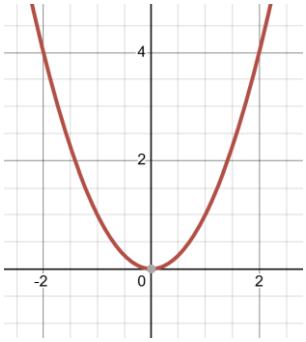
$$\angle A + \angle B = 180°$$



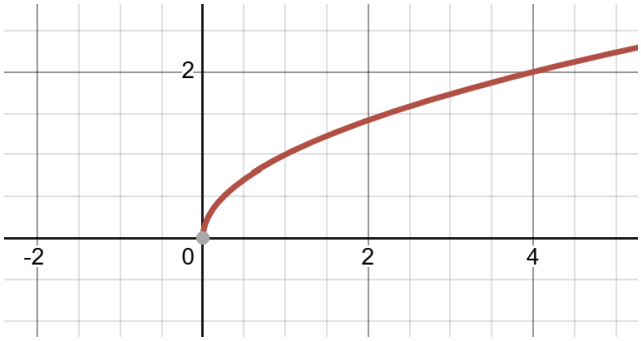
Polynomial Functions

Shapes

$f(x) = x^2$



$f(x) = \sqrt{x}$



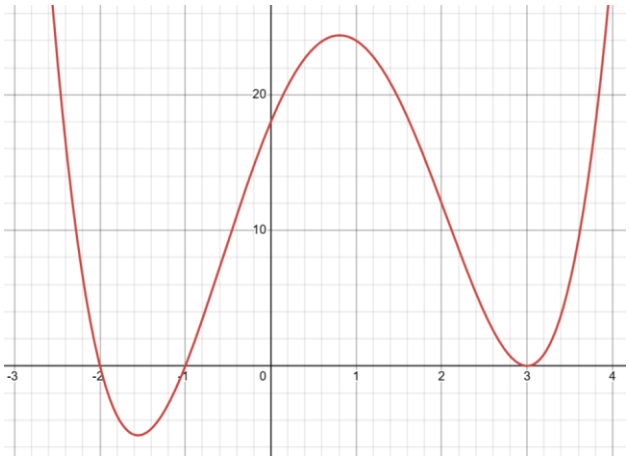
Remainder Theorem

If a polynomial $P(x)$is divided by $(x-a)$, the remainder is $P(a)$.

Factor Theorem

If $P(a) = 0$, then $(x-a)$is a factor of $P(x)$.

Drawing Polynomial Functions: $P(x) = a(x - r_1)^{m_1}(x + r_2)^{m_2}(x + r_3)^{m_3}$



y-intercept: Occurs where $x = 0$

x-intercepts (roots): Occur where $y = 0$

Finding roots from factors:

$(x - r) = 0 \Rightarrow x = r$

Leading coefficient a: Controls vertical stretch/compression and reflection

Multiplicity: Repeated factors

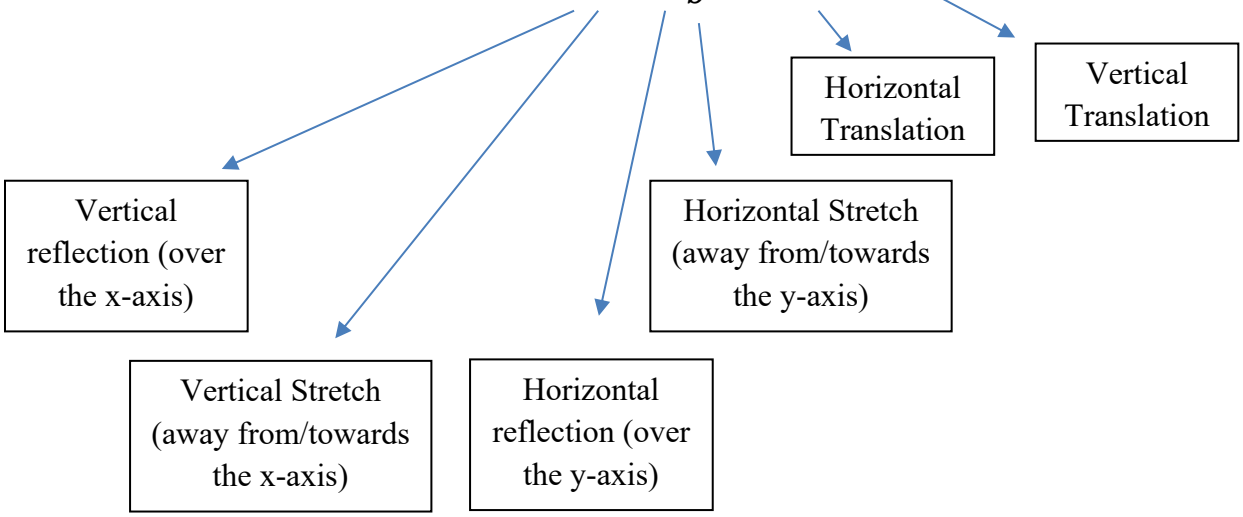
even multiplicity → touch,
odd multiplicity → cross)

End Behavior

- Even degree, positive leading coefficient: both ends ↑
- Even degree, negative leading coefficient: both ends ↓
- Odd degree, positive leading coefficient: left ↓, right ↑
- Odd degree, negative leading coefficient: left ↑, right ↓

Function Transformations

$$y = \pm af\left(\pm\frac{1}{b}(x-h)\right)+q$$



Inverses

$$x = f(y)$$

Steps to find:

1. Swap $x$and y
2. Solve for y

Absolute Functions

$$g(x) = |f(x)|$$

A conditional reflection over the x-axis.

Root Functions

$$g(x) = \sqrt{f(x)}$$

A transformation from a quadratic function to a linear function to a root function.

Reciprocal Functions

$$g(x) = \frac{1}{f(x)}$$

A transformation from a large values to small values and from small values to large values.

Exponents & Logarithms

Laws of Exponents

$$a^x a^y = a^{x+y} \qquad a^y b^y = (ab)^y$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad a^0 = 1,$$

$$(a^m)^n = a^{mn} \qquad a^{-n} = \frac{1}{a^n}$$

Radical Properties

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \qquad a\sqrt{b} = \sqrt{a^2 b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Laws of Logarithms

$$log_a(x) = y \iff a^y = x$$

$$log\, a^m = m\, log\, a$$

$$log_a(1) = 0, \qquad log(ab) = log\, a + log\, b$$

$$\log_a(a) = 1 \qquad log\left(\frac{a}{b}\right) = log\, a - log\, b$$

$$log_a\, b = \frac{log\, b}{log\, a}$$

Natural Logarithm

$$e^{\ln x} = x \qquad \ln 1 = 0,$$

$$\ln e^x = x \qquad \ln\, e = 1$$

$$\ln x = log_e x,$$

Arithmetic Sequences

$$t_n = t_1 + (n-1)d \qquad S_n = \frac{n(t_1 + t_n)}{2}$$

Geometric Sequences

$$\mathrm{t_n} = \mathrm{ar^{n-1}} \qquad r = \frac{\mathrm{t_2}}{\mathrm{t_1}}$$

Applications of Geometric Sequences

Compound Interest → $A = P\left(1+\frac{r}{n}\right)^{nt}$

Population → $P_f = P_i r^{\boldsymbol{periods}}$

Half Lives → $m_f = m_i\left(\frac{1}{2}\right)^{\frac{t}{half-life}}$

Investment → $A = P(r)^{\boldsymbol{periods}}$

Earthquake Intensity → $I = 10^{R_F - R_o}$

pH → $[H^+] = (10)^{-pH}$

Sound Intensity → $I = 10^{\frac{D_f - D_o}{10}}$

Arithmetic Sequences

$$a_n = \mathrm{a} + d(n-1)$$

$$S_n = \frac{n(\mathrm{a} + t_n)}{2}$$

Geometric Series

$$S_n = a\frac{(1-r^n)}{1-r}$$

$$S_{n+1} - S_n = \mathrm{t_{n+1}}$$

Infinite Geometric Series

$$S_\infty = a\left(\frac{1}{1-r}\right)$$

Ordinary Annuity

$$Sn = A\frac{(1+r)^n - 1}{r}$$

Annuity Due

$$Sn = A(1+r)\frac{(1+r)^n - 1}{r}$$

Continuous Growth Rate

$$e^r = \left(1+\frac{r}{n}\right)^n$$

$$A = P(e^r)^t$$

$$S_n = Ae^r\left(\frac{e^{rn}-1}{e^r-1}\right)$$

Trigonometry Formula Sheet

Angles in Standard Position

Vertex at origin & Initial arm on positive x-axis

Reference Angle

Acute angle between terminal arm and x-axis

Coterminal Angles

$$\theta = \theta + 360°k$$

Angle Conversion

$$\frac{Radians}{Degrees} = \frac{2\pi}{360}$$

Function Transformations

$$y = A\,sin\big(B(x - C)\big) + D \quad \text{and} \quad y = A\,cos\big(B(x - C)\big) + D \quad \text{where} \quad B = \frac{2\pi}{p}$$

Parameters

Amplitude: $|a|$ Phase shift: c

Period: $\frac{2\pi}{|b|}$ Vertical shift: d

Right Triangle Ratios (SOH-CAH-TOA)

$$sin\,\theta = \frac{o}{h} \qquad cos\,\theta = \frac{a}{h} \qquad tan\,\theta = \frac{sin\,\theta}{cos\,\theta}$$

$$csc\,\theta = \frac{1}{sin\,\theta} \qquad sec\,\theta = \frac{1}{cos\,\theta} \qquad cot\,\theta = \frac{1}{tan\,\theta}$$

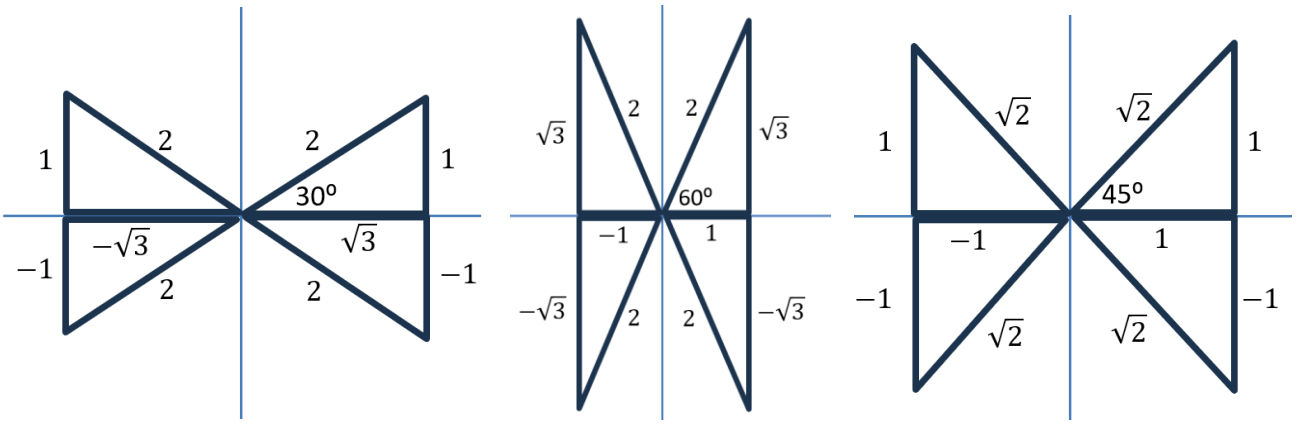
Cosine Law

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Sine Law

$$a\sin B = b\sin A \qquad \frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{\sin A}{a} = \frac{\sin B}{b}$$

Special Triangles & Bowties



Trigonometric Identities

$$sin^2\theta + cos^2\theta = 1$$

$$tan^2\theta + 1 = sec^2\theta$$

$$1 + cot^2\theta = \csc^2\theta$$

$$1 + cos\,\theta \quad OR \quad 1 - cos\,\theta \quad \rightarrow \quad Multiply\ by\ Conjugate$$

$$1 + sin\,\theta \quad OR \quad 1 - sin\,\theta \quad \rightarrow \quad Multiply\ by\ Conjugate$$

Sum & Difference Angle Identities

Sine:

$$sin(A + B) = sin(A)cos(B) + cos(A)sin(B)$$

$$sin(A - B) = sin(A)cos(B) - cos(A)sin(B)$$

Cosine:

$$cos(A + B) = cos(A)cos(B) - sin(A)sin(B)$$

$$cos(A - B) = cos(A)cos(B) + sin(A)sin(B)$$

Tangent:

$$tan(a + b) = \frac{tan\,a + tan\,b}{1 - tan\,a\,tan\,b}$$

$$tan(a - b) = \frac{tan\,a - tan\,b}{1 + tan\,a\,tan\,b}$$

Double Angle Identities

Sine:

$$sin(2A) = 2sin(A)cos(A)$$

Cosine:

$$cos(2A) = cos^2(A) - sin^2(A)$$

$$cos(2A) = 2cos^2(A) - 1$$

$$cos(2A) = 1 - 2sin^2(A)$$

Tangent:

$$tan(2A) = \frac{2tan(A)}{1 - tan^2(A)}$$