Function Reciprocals

1. For each function:
   a. Sketch $f(x)$
   b. Find the zeros of $f(x)$
   c. Determine where $f(x) > 0$and $f(x) < 0$
   d. Sketch $\frac{1}{f(x)}$

$$y = \frac{1}{(x-3)^2-4} \qquad y = \frac{1}{3(x+2)^2 - 5}$$

$$y = \frac{1}{-2(x-1)^2 + 6} \qquad y = \frac{1}{(x-2)(x+4)}$$

2. If $f(x)$has two distinct zeros at $x = a$and $x = b$, what happens to those x-values near 0 on $y = \frac{1}{f(x)}$?

3. If $f(x)$ has exactly one zero, what happens to those x-values near 0 on $y = \frac{1}{f(x)}$?

4. If $f(x)$never equals zero, what must be true about the graph of $\frac{1}{f(x)}$?

5. If a quadratic opens upward and has minimum value $m < 0$, describe the range of its reciprocal.

6. If a quadratic opens downward and has maximum value $M > 0$, describe the range of its reciprocal.

7. If the vertex of a quadratic lies on the x-axis, what special feature does the reciprocal have? How is this different from the case of two zeros?

8. How does the value (positive or negative) of $f(x)$on an interval determine the position of the reciprocal branch on that interval?

9. If $f(x) \geq 0$for all x, what can you conclude about the reciprocal? Under what condition would it be undefined?

10. Suppose $f(x)$is symmetric about $x = h$. Is its reciprocal also symmetric about $x = h$? Why?

11. If $f(x)$is very close to zero but not equal to zero, what happens to the reciprocal? What does this explain about asymptotes?

12. If two quadratics have the same zeros but different vertical stretch factors, how do their reciprocals compare structurally?

13. Can $\frac{1}{f(x)}$ever have more zeros than $f(x)$? Explain carefully.

14. For each function, determine:

- domain
- x-intercepts
- y-intercept
- vertical asymptotes
- horizontal asymptote

a. $f(x) = \frac{1}{x-3}$ b. $f(x) = \frac{1}{x-2}$ c. $f(x) = \frac{1}{x+4}$

15. Determine whether the graph contains a **hole or vertical asymptote**.

- factor
- simplify
- identify holes

a. $f(x) = \frac{x^2-9}{x-3}$ b. $f(x) = \frac{x^2-4}{x^2-5x+6}$ c. $f(x) = \frac{x+3}{(x+3)(x-1)}$

16. Determine the horizontal asymptote. Explain **why** the asymptote occurs.

a. $f(x) = \frac{1}{x+1}$ b. $f(x) = \frac{1}{x^2-5}$ c. $f(x) = \frac{1}{2x^2-4}$

17. Analyze completely and sketch. $f(x) = \frac{1}{x^2-4}$ Find:

- domain
- intercepts
- vertical asymptotes
- horizontal asymptote
- holes
- end behaviour

18. If $f(x)$has a zero at $x = a$, explain why the reciprocal must have a vertical asymptote at $x = a$.