

Proportional Relationships

1. Consider the equation for the gravitational force between two masses where, $F_g = \frac{Gm_1m_2}{r^2}$

Two objects are separated by a distance of 1000 km (from their centres). The gravitational force at this distance is 500 N. Determine the gravitational force between the masses for the following changes.

- One mass is doubled.
 - Both masses are tripled.
 - The distance separating the masses is increased to 5000 km.
 - The distance separating the masses is decreased to 250 km.
 - The distance separating the masses is increased to 1250 km.
 - One mass is increased by a factor of five and the distance is increased to 2000 km.
 - Both masses are halved and the distance separating the masses is halved.
 - One mass is tripled, the other is halved, and the distance separating them is decreased to 800 km.
2. Consider the equation for kinetic energy where, $E_k = \frac{1}{2}mv^2$

m represents the mass

v represents the velocity.

Determine the change in the kinetic energy for each of the following changes.

- The mass is doubled.
 - The velocity is doubled.
 - The mass is halved.
 - The velocity is halved.
 - The mass and velocity are both increased by a factor of three.
 - The mass and velocity are both decreased by a factor of three.
 - The mass is doubled and the velocity is halved.
 - The mass is decreased by a factor of four and the velocity is doubled.
3. For each pair of variables, determine the relationship (rearrange the equation if necessary). The relationships should be a description of how the first variable depends on the second.

a. area, A , and radius, r (for a circle.) $A = \pi r^2$

b. volume, V , and radius, r (for a sphere). $V = \frac{4\pi r^3}{3}$

c. displacement, d , and velocity, v (uniform motion). $d = vt$

d. displacement, d , and time, t (accelerating from rest). $d = v_i t + \frac{1}{2}at^2$

e. acceleration, a , and net force, F_{NET} . $F_{\text{net}} = ma$

f. acceleration, a , and mass, m . $F_{\text{net}} = ma$

g. force of friction, F_f , and normal force, F_N . $F_f = \mu F_N$

- h. displacement from equilibrium position, Δx , and spring constant, k . $F_s = k\Delta x$
- i. gravitational field strength, g , and mass, M . $g = \frac{Gm}{r^2}$
- j. gravitational force, F_g , and separation distance, r . $F_g = \frac{Gm_1m_2}{r^2}$
- k. momentum, p , and velocity, v . $p = mv$
- l. gravitational potential energy, E_p , and height, h . $E_p = mgh$.
- m. kinetic energy, E_k , and mass, m . $E_k = \frac{1}{2}mv^2$
- n. kinetic energy, E_k , and velocity, v . $E_k = \frac{1}{2}mv^2$
- o. power, P , and time, t . $P = \frac{W}{t}$
- p. current, I , and resistance, R . $P = I^2R$

4. Determine the change in the following variables.

- a. area, A , if radius, r , is doubled (for a circle). $A = \pi r^2$
- b. volume, V , if radius, r , is halved (for a sphere). $V = \frac{4\pi r^3}{3}$
- c. displacement, d , if velocity, v , increased by a factor of five (uniform motion). $d = vt$
- d. acceleration, a , if net force, F_{NET} , is three times its original value. $F_{\text{net}} = ma$
- e. acceleration, a , if mass, m , is doubled. $F_{\text{net}} = ma$
- f. force of friction, F_f , if normal force, F_N , is one-third its original value. $F_f = \mu F_N$
- g. displacement from equilibrium position, Δx , if spring constant, k , is halved. $F_s = k\Delta x$
- h. gravitational field strength, g , if mass, M , is increased by a factor of four. $g = \frac{Gm}{r^2}$
- i. gravitational force, F_g , if separation distance, r , is increased by a factor of six. $F_g = \frac{Gm_1m_2}{r^2}$
- j. momentum, p , if velocity, v is 2.5 times its original value. $p = mv$
- k. gravitational potential energy, E_p , if height, h , is decreased by a factor of ten. $E_p = mgh$
- l. kinetic energy, E_k , if mass, m , is halved. $E_k = \frac{1}{2}mv^2$
- m. kinetic energy, E_k , if velocity, v , is halved. $E_k = \frac{1}{2}mv^2$
- n. power, P , if time, t , is twice its original value. $P = \frac{W}{t}$
- o. current, I , if resistance, R , is $4/5$ its original value. $P = I^2R$

5. Consider the equation for the volume of a cylinder where

$$V = \pi r^2 h$$

r represents the radius

h represents the height.

- Determine the proportional relationship between volume, V , and radius, r .
- Determine the proportional relationship between volume, V , and height, h .

Determine the change in volume for each of the following changes:

- The height is increased by a factor of four.
- The radius is halved.
- The radius is decreased by a factor of three and the height is doubled.

A cylindrical glass can hold 400 mL of water. Determine how much water the glass can hold for each of the following changes.

- The height is tripled.
- The radius is doubled.
- The radius is halved and the height is decreased by a factor of four.

6. Consider the equation for magnetic field around a current-carrying wire where

$$B = \frac{\mu_0 I}{2\pi d}$$

μ_0 represents the permeability of free space,

I represents the current through the wire,

d represents distance from the wire.

- Determine the proportional relationship between magnetic field, B , and current, I .
- Determine the proportional relationship between magnetic field, B , and distance, d .

Determine the change in magnetic field for each of the following changes.

- The current is halved.
- The distance from the wire is decreased by a factor of five.
- The current is increased by a factor of ten and the distance is tripled.

A long wire carries a current of 100 mA. At a distance x from the wire, the magnetic field is found to be 20 nT. Determine the magnetic field for each of the following changes.

- The current is decreased to 25 mA.
- The distance from the wire is increased to $5x$.
- The current is increased to 300 mA and the distance from the wire is decreased to $x/4$.

7. Consider the equation for the period of a mass-spring oscillator where

$$T = 2\pi \sqrt{\frac{m}{k}}$$

m represents the mass

k represents the spring constant.

- Determine the relationship between period, T , and mass, m .
- Determine the relationship between period, T , and the spring constant, k .

Determine the change in period for each of the following changes.

- The mass is decreased by a factor of four.
- The spring constant is increased by a factor of nine.
- The mass and spring constant are both tripled.

A mass attached to a spring oscillates with a period of 0.80 seconds. Determine the period for each of the following changes.

- The spring constant is decreased by a factor of four.
- The mass is increased by a factor of 25.
- The mass is halved and the spring constant is increased by a factor of eight.

8. Consider the equation for the electric force between two charges where

$$F_e = k \frac{q_1 q_2}{r^2}$$

k represents the electrostatic constant,

q_1 and q_2 represent the charges

r represents the separation distance.

Two charges are separated by a distance of 20 mm. The electric force at this distance is 2 N. Determine the electric force between the charges for the following changes.

- One charge is halved.
- Both charges are increased by a factor of three.
- The distance separating the charges is increased to 100 mm.
- The distance separating the charges is decreased to 10 mm.
- The distance separating the charges is decreased to 50 mm.
- One charge is halved and the distance separating the charges is decreased to 10 mm.
- Both charges are increased by a factor of ten and the distance separating the masses is increased to 100 mm.
- One charge is doubled, the other is decreased by a factor of five, and the distance separating them is decreased to 4 mm.