Study Guide: Polynomials & Transformations

POLYNOMIALS

---

1. $P(x) = -2(x-1)^2(x+4)$

a) Roots

b) Multiplicities

c) Degree

d) End behaviour

e) y-intercept

f) Sketch

---

2. $P(x) = 3(x+2)^3(x-5)$

a) Roots

b) Multiplicities

c) End behaviour

d) Sketch

---

3. Factor Completely $P(x) = x^3 - 2x^2 - 15x$

Then sketch.

---

4. Factor Completely $P(x) = -2x^3 + 5x^2 + 12x$

Then sketch.

---

5. Behaviour Description

A polynomial:

• Crosses at −3

• Touches at 2

• Flattens and crosses at 4

a) Multiplicities

b) Minimum degree

c) Leading coefficient sign if graph falls right

---

6. Behaviour Description

A polynomial:

• Touches at −1

• Crosses at 3

• Crosses at 6

Graph rises left and right.

a) Multiplicities

b) Minimum degree

c) Leading coefficient sign

---

7. Conceptual

True or False. Justify.

a) A multiplicity of 5 must cross.

b) An even degree polynomial must have both ends in same direction.

c) A cubic must have at least one real zero.

---

8. Conceptual

If a polynomial has exactly one real root and degree 4, what must be true about that root?

---

9. Factor Theorem

$$P(x) = x^3 - 7x + 6$$

---

10. Factor Theorem

$$P(x) = x^3 - 4x^2 - 7x + 10$$

---

11. Remainder Theorem

When $x^3 - kx^2 + 4x - 3$ is divided by $x - 1$, remainder is 8. Find k.

---

12. Remainder Theorem

When $x^3 + 2x^2 - kx + 5$ is divided by $x - 2$, remainder is 3. Find k.

---

13. Construct a Polynomial

Zeros: −2, 1, 4

y-intercept: −16

---

14. Construct a Polynomial

Zeros: 3 (double root), −5

y-intercept: 90

---

15. Modelling

Length = 2h

Width = h + 6

Height = h – 1

Volume = 180

Solve for h.

---

16. Modelling

Length = h

Width = h + 4

Height = h + 2

Volume = 240

Solve for h.

TRANSFORMATIONS

---

17. List transformations in correct order: $y = -3f(2(x-4)) + 5$

---

18. List transformations in correct order: $y = \frac{5}{2}f(-4(x+1)) - 3$

---

19. Quadratic

Start from $y = x^2$:

• Reflect over x-axis

• Horizontal compression by 3

• Shift left 2

• Shift up 4

Write final equation and sketch.

---

20. Quadratic

Start from $y = x^2$:

• Vertical stretch by 2

• Horizontal stretch by ½

• Shift right 5

• Shift down 1

Write final equation and sketch.

---

21. Radical

Start from $y = \sqrt{x}$:

• Vertical stretch 4

• Horizontal stretch 2

• Shift right 3

• Shift down 5

Write equation, state domain, sketch.

---

22. Radical

Start from $y = \sqrt{x}$:

• Reflect over x-axis

• Horizontal compression by 5

• Shift left 1

• Shift up 6

Write equation and sketch.

---

23. Coordinate Mapping

(6, 2) lies on $f(x)$.

Find image under:

$$g(x) = -2f(3(x+1)) - 4$$

---

24. Coordinate Mapping
$(4, -3)$ lies on $f(x)$.
Find image under:

$$g(x) = 3f\left(\frac{1}{2}(x-2)\right) + 7$$

---

25. Linear Inverse

$$y = 5 - \frac{1}{2}x$$

---

26. Linear Inverse

$$y = \frac{3}{4}x - 8$$

---

27. Quadratic Relation Inverse

$$x = 2(y-3)^2 + 1$$

Rewrite as $y = \cdots$

---

28. Quadratic Relation Inverse

$$x = -3(y+2)^2 + 5$$

Rewrite as $y = \cdots$

---

29. Absolute Value

$$f(x) = (x-1)(x+3)$$

Sketch $f(x)$and $|\ f(x)\ |$.

---

30. Absolute Value

$$f(x) = -2(x-4)(x+2)$$

Sketch $f(x)$and $|\ f(x)\ |$.

---

31. Structural Insight
If a quadratic opens upward and has minimum -8:
a) Minimum of $|\ f(x)\ |$?
b) Explain.

---

32. Zero Behaviour
If $f(a) = 0$, determine whether $a$is still a zero of:
a) $f(x) + 4$
b) $2f(x)$
c) $f(x-4)$
d) $-f(x)$