Solving Wave Problems

Using distance: $y = A\sin\left(\frac{2\pi}{\lambda}x\right)$ and using time: $y = A\sin\left(\frac{2\pi}{\mathrm{T}}x\right)$

1) A small mass on a spring oscillates with an amplitude of **2 cm**. It completes one full cycle every **3 seconds**, returning to its starting position.
   a) Write a sine function for the motion, assuming the mass passes through y=0 going upward at t=0.
   b) Determine the earliest time t > 0 when the mass first reaches y=1 cm (half its amplitude).

2) A child's swing moves up to **20 cm** from its center. One full back-and-forth swing takes **4 seconds**.
   a) Write the displacement wave function of the swing from the midpoint, using a sine function with period T=4 s.
   b) How many times does the child reach the maximum amplitude in the first 12 seconds?

3) A small toy boat on a pond bobs up and down with an amplitude of **5 cm**. It completes **10 oscillations** in **20 seconds**.
   a) Calculate the frequency f and the period T.
   b) Write the vertical motion as a wave function.
   c) Find the first positive time when the boat's displacement is 2.5 cm (half its amplitude).

4) A tuning fork, when struck, produces small vertical oscillations of **0.05 cm** amplitude on a device that measures displacement. The fork vibrates at **512 Hz** (cycles per second).
   a) Determine the period T of the tuning fork's vibration.
   b) Write y(t), assuming the fork passes through y=0 heading upward at t=0.
   c) How many complete oscillations occur in the first **0.01 seconds**?

5) A light earth tremor causes a seismograph to oscillate with a **1 mm** amplitude. Each full vibration repeats every **5 seconds**.
   a) Write the wave function with amplitude 1 mm and period 5 s.
   b) At what times between t=0 and t=10 s is the seismograph's displacement exactly zero?

6) A different mass-spring setup has an amplitude of **3 cm**. It completes **2.5 cycles** each second (f=2.5 Hz).
   a) Find the period T from the given frequency f.
   b) Write y(t) for this oscillation (assume it crosses y=0 going upward at t=0).
   c) In the first **2 seconds**, how many times does it reach y=0?

7) A ripple on a shallow pond has an amplitude of **3 cm** and a wave speed of **0.6 m/s**. Its crests pass by at a rate of **1.5 Hz**.
   a) Calculate the wavelength.
   b) Write the wave profile y(x) for a **snapshot in time** (assume no phase shift at x=0).
   c) Find the **smallest positive** x for which y(x)=1.5 cm.

8) A skipping rope is held tight so that bumps of amplitude **5 cm** form along it. The rope's wave speed is **1.2 m/s**, and the rope is shaken at **2 Hz**.
   a) Compute the wavelength.
   b) Write the spatial wave function at a fixed instant.
   c) If x=0 is at a the centre of the wave initially, how far until the next x>0 position where the rope is at y=0.

9) A low-frequency sound wave has a **pressure amplitude** of **0.5 Pa**. It travels through air at **340 m/s** and vibrates at **256 Hz**.
   a) Determine the wavelength.
   b) Write the spatial wave function.
   c) Find the **first positive** x where p(x)=0.25 Pa (half the amplitude).

10) A wave travels along a water trough with amplitude **2 cm** at a speed of **0.4 m/s**. It completes **4 cycles** every second.
   a) Calculate the wavelength from the wave speed v and frequency f.
   b) Write the spatial function y(x).
   c) If x=0 is at equilibrium (y=0), where is the **first crest** along the positive x-axis?

11) An infrared light wave has an electric field amplitude of **3 V/m**. It travels in vacuum ($c=3\times10^8$ m/s) at a frequency of $\mathbf{6\times10^{14}}$.
   a) Compute the wavelength.
   b) Write the electric field wave function.
   c) How far from x=0 (the first point of zero crossing) must you move to find the wave at **peak amplitude**?

12) A string is producing stationary "bumps" (like standing waves) of amplitude **1 cm**. The distance between two adjacent crests is **20 cm**.
   a) From the crest-to-crest distance, identify the wavelength.
   b) Write a **spatial** sine function y(x) that has its first crest at x=10 cm.
   c) If you stand at x=15 cm, what is y there?