HOUSE

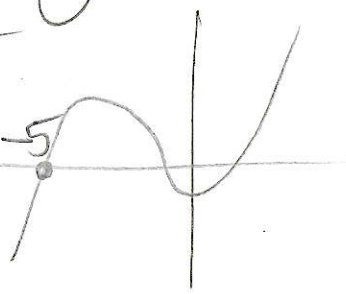
Polynomial Function Problems

1. Which of the following binomials is a factor of the polynomial $P(x) = 2x^3 + 5x^2 - 28x - 15$

A. $2x-3$ B. $x+3$ C. $2x-1$ D. $x+5$

$2(-3)^3 + 5(-3)^2 - 28(-3) - 15$

$-54 + 45 + 84 - 15 \neq 0$



2. In a polynomial $P(x)$, $P\left(-\frac{2}{5}\right) = 0$. One of the factors of $P(x)$ is

A. $(5x-2)$ B. $(2x-5)$ C. $(5x+2)$ D. $(2x+5)$

$x = -\frac{2}{5}$

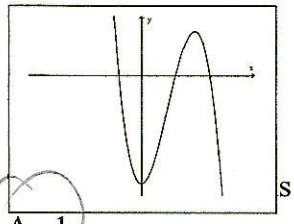
$5x = -2$

$5x+2=0$

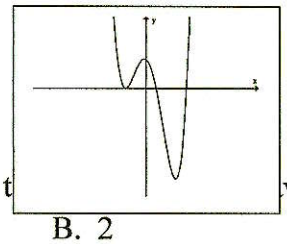
3. If $x+2$ is a factor of P(x), then the value of P(-2) is:

A. 0 B. $-\frac{1}{2}$ C. 2 D. -2

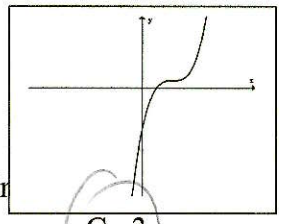
4. Sam is presented with the following four partial graphs of polynomial functions, and is asked to select the graphs with an odd degree.



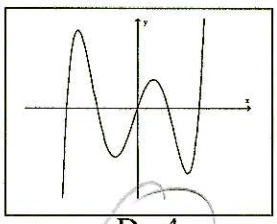
A. 1



B. 2



C. 3



D. 4

5. The remainder when $P(y) = 2y^4 - 3y^2 + 1$ is divided by $(y-3)$ is _____.

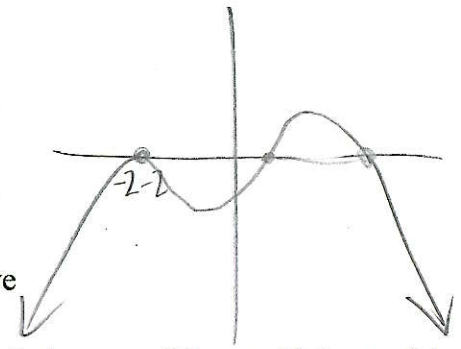
$P(3) = 2(3)^4 - 3(3)^2 + 1$

$= 162 - 27 + 1$

$= 136$

6. Sketch a graph of a polynomial function with the following characteristics, using the grid below.

a) Degree of four
b) Zeroes: -2 with a multiplicity of two
1 with a multiplicity of one
3 with a multiplicity of one
c) y-intercept of -3
d) end behaviour on the right is negative



$+0x^4$ $+0x$

7. When $2x^5 - 3x^3 - 8x^2 - 8$ is divided by $x - 2$, the sum of the coefficients of the terms in the quotient is

A. –59
B. –23
C. –19
D. 17

$2x^4 + 4x^3 + 5x^2 + 2x + 4$

	$2x^5$	$4x^4$	$+5x^3$	$+2x^2$	$4x$
x					
-2	$-4x^4$	$-8x^3$	$-10x^2$	$-4x$	-8

$2+4+5+2+4$
$= 17$

$+0x^2$

8. If $P(x) = x^3 - 7x - 3$ is divided by $(x + 1)$, then $P(x)$ may be written as

A. $(x^2 - 8)(x + 1) + 5$
B. $(x^2 - x - 6)(x + 1) + 3$
C. $(x^3 - 7x - 3)(x + 1) + 3$
D. $(x^3 - 7x - 3)(x + 1) + 5$

$x^2 - 1x - 6$

	x^3	$-1x^2$	$-6x$
x			
$+1$	$1x^2$	$-1x$	-6

$+3$

$(x^2 - 1x - 6)(x + 1) + 3$

9. If $-5x$ is a factor of the polynomial $P(x)$, then $P(0)$ is

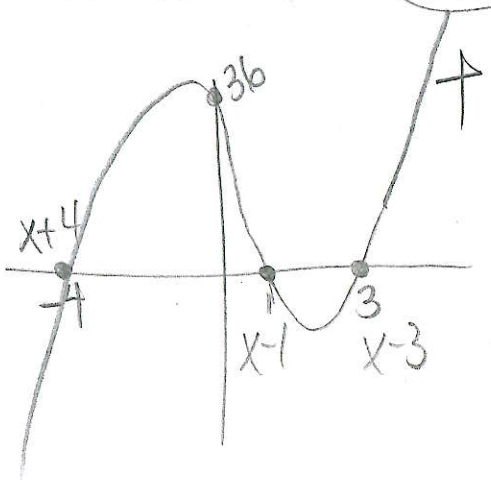
A. –5
B. 0
C. $\frac{1}{5}$
D. 5

$P(x) = -5x(x+2)(x-4)^2$

$P(0) = 0$

10. If $P(x)$ is a cubic polynomial function with $P(1) = P(3) = P(-4) = 0$ and $P(0) = 36$, then $P(x)$ is

A. $-3(x - 1)(x - 3)(x + 4)$
~~B. $-2(x + 1)(x + 3)(x - 4)$~~
~~C. $2(x + 1)(x + 3)(x - 4)$~~
D. $3(x - 1)(x - 3)(x + 4)$



11. If the polynomial function $P(x) = 3x^3 - 9x^2 + kx - 12$ is divisible by $x - 3$, then it is also divisible by

A. $3x - 4$
B. $3x + 4$
C. $3x^2 + 4$
D. $3x^2 - 4$

$x = 3$

$3(3)^3 - 9(3)^2 + k(3) - 12 = 0$

$k = 4$

$3x^2 + 0x + 4$

	$3x^3$	$0x^2$	$+4x$
x			
-3	$-9x^2$	$0x$	-12

12. The area of a rectangle is $(x^3 + 5x^2 - 4x - 20)$ cm^2 and the width is $(x + 2)$ cm. If the length of the rectangle is 8 cm, then the width in centimetres is

A. 3 cm
B. 5 cm
C. 6 cm
D. 8 cm

$x^2 + 3x - 10$

	x^3	$3x^2$	$-10x$
x			
$+2$	$2x^2$	$6x$	-20

$x^2 + 3x - 10 = 8$

$x^2 + 3x - 18 = 0$

$(x + 6)(x - 3) = 0$

$x = -6$ or $x = 3$

13. If $(x - 3)$ is a factor of the polynomial function P, where $P(x) = x^3 - 2x^2 + kx + 6$, then the value of k is

$x = 3$ is a root

A. -13
B. -5
C. 5
D. 13

$(3)^3 - 2(3)^2 + k(3) + 6 = 0$

$27 - 18 + 3k + 6 = 0$

$3k + 15 = 0$

$k = -5$

14. An integral polynomial function is defined by $P(x) = x^3 - ax^2 - 2x + 2a$, $a \neq 0$. Which of the following is a factor of $P(x)$ for any value of a?

A. $x - a$
B. $x + \sqrt{a}$
C. $x - \sqrt{a}$
D. $x + a$

If $x = a$ is a root then $a^3 - a(a^2) - 2(a) + 2a = 0$ ✓

15. The polynomial function $P(x) = 4x^3 - 18x^2 - x - 45$ is divided by $D(x) = x - 5$. The quotient is of the form $Q(x) = ax^2 + bx + c$. Determine the values of a, b, and c, where a, b, and c.

$4x^2 + 2x + 9$

	$4x^3$	$+2x^2$	$+9x$
x			
-5	$-20x^2$	$-10x$	-45

$a = 4$

$b = 2$

$c = 9$

16. The polynomial $f(x)=x^3-x^2-40x+n$ is divisible by $x-3$. The value of n, correct to the nearest whole number, is _____.

If $x=3$ is a root

then $(3)^3-(3)^2-40(3)+n=0$

$27-9-120+n=0$

$n=102$

17. A factor of $4x^3-8x^2+5x-1$ is $x-1$. The other first degree factors of this polynomial are

a. $(2x-1)$ *and* $(2x+1)$

b. $(2x-1)$ *and* $(2x-1)$

c. $(4x-1)$ *and* $(x+1)$

d. $(x-1)$ *and* $(x+1)$

$4x^2-4x+1$

x	$4x^3$	$-4x^2$	$+1x$
-1	$-4x^2$	$+4x$	-1

$2x-1$

$2x$	$4x^2$	$-2x$
-1	$+2x$	$+1$

18. A step in the process of factoring the polynomial $P(x)=x^5-x^3+2x-2$ is

a. $(x-1)(x^4-2x^2)$

b. $(x-1)(x^4+x^3+2)$

c. $(x+1)(x^4-x^3+2)$

d. $(x+1)(x^4-x^3-2x^2-2x-4)$

$+0x^4$

x^4+x^3+0+2

x	x^5	$1x^4$	$0x^2$	$2x$
-1	$-x^4$	$-x^3$	0	-2

19. For the polynomial $P(x)=3x^3-2x^2-7x-2$, two of the zeros are 1 and 2. The third zero for this polynomial is

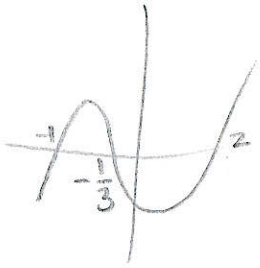
a. $\frac{5}{3}$

b. $\frac{2}{3}$

c. $\frac{1}{3}$

d. $-\frac{1}{3}$

$x=-\frac{1}{3}$



20. A student graphed the equation of a polynomial $y=P(x)$ to help determine the solution to the equation $P(x)=0$. Which of the following provides the solution to $P(x)=0$.

a. The x intercepts of the graph of $y=P(x)$

b. The y intercepts of the graph of $y=P(x)$

c. The point(s) of intersection of $x=0$ and the graph of $y=P(x)$

d. The point(s) of intersection of $y=0$ and the graph of $P(0)$

21. Which of the following functions is a polynomial function?

a. $f(x)=2^x+6x^2+5$

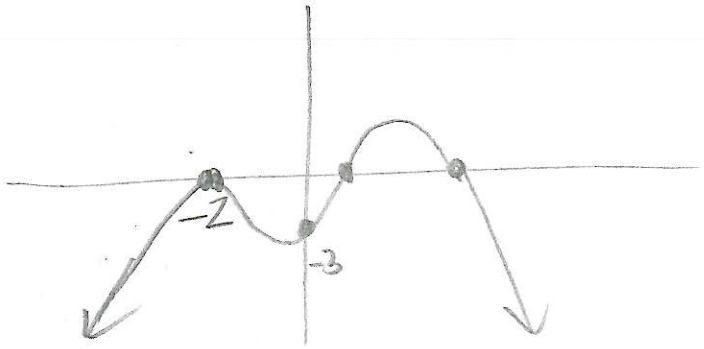
b. $f(x)=x^2+6x+\sqrt{5}$

c. $f(x)=x^{\frac{2}{3}}+5x^2-7$

d. $f(x)=\frac{x^3}{3}+\sqrt{5}x-\frac{1}{x}$

22. Sketch a graph of a polynomial function with the following characteristics, using the grid below.

a) Degree of four
b) Zeroes: -2 with a multiplicity of two
1 with a multiplicity of one
3 with a multiplicity of one
c) y-intercept of -3
d) end behaviour on the right is negative



23. The polynomial $f(x)=x^3-7x^2+2x+n$ is divisible by $x+2$. The value of n, correct to the nearest whole number, is 40.

$x=-2$ is a root

$(-2)^3-7(-2)^2+2(-2)+n=0$

$-8-28-4+n=0$

$n=40$

24. A factor of $4x^3-x^2-4x+1$ is $x-1$. The other first degree factors of this polynomial are

a. $(2x-1)and(2x+1)$
b. $(2x-1)and(2x-1)$
c. $(4x-1)and(x+1)$
d. $(x-1)and(x+1)$

$x=-1$ $x=\frac{1}{4}$ $x=1$
$x+1$ $4x=1$ $x-1$
$4x-1=0$

25. A student graphed the equation of a polynomial $y=P(x)$ to help determine the solution to the equation $P(x)=0$. Which of the following provides the solution to $P(x)=0$.

a. The point(s) of intersection of $x=0$ and the graph of $y=P(x)$
b. The point(s) of intersection of $y=0$ and the graph of $P(0)$
c. The x intercepts of the graph of $y=P(x)$
d. The y intercepts of the graph of $y=P(x)$

26. The factors of the polynomial $P(x) = 6x^3 + x^2 - 10x + 3$ are $(2x+3), (3x-1)$, and $(x-1)$. If the polynomial $P(x)$ is multiplied by -2, then the result is the polynomial $-2P(x)$. The zeroes of the new function are

$x = -1.5 \quad x = \frac{1}{3} \quad x = 1$

a) $\frac{-3}{2}, \frac{1}{3}, 1$

b) $\frac{3}{2}, -\frac{1}{3}, -1$

c) $-3, \frac{2}{3}, 2$

d) $3, -\frac{2}{3}, -2$

27. Which statement is true?

a. Some odd degree polynomial functions have no x-intercepts
b. Even-degree polynomial functions always have an even number of x-intercepts
c. All odd-degree polynomial functions have at least one x-intercept
d. All even degree polynomial functions have at least one x-intercept

28. A student graphed the equation of a polynomial function, $y = P(x)$, to help determine the solution to the equation $P(x) = 0$. Which of the following provides the solution to $P(x) = 0$?

A. The x-intercepts of the graph of $y = P(x)$.
B. The y-intercept of the graph of $y = P(x)$.
C. The point(s) of intersection of $x = 0$ and the graph of $y = P(x)$.
D. The point(s) of intersection of $y = 0$ and $P(0)$.

29. The equation of a polynomial function is $P(x) = kx(x+2)(x-6)$, $k \neq 0$. If a new function results from doubling the value of k only, then the zeros of the new function

A. remain unchanged
B. are doubled
C. are 0, –1, and 3
D. are 0, –4, and 12

K is in the 'a' spot doubling will not change the x-intercepts

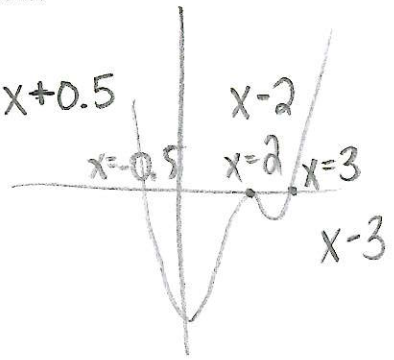
30. Circle the polynomial functions below. Explain why the other(s) are not polynomial functions.

i. $y = \frac{3}{4}x^{(-5)} - 1$	ii. $y = 3x^4 - x^{\left(\frac{1}{2}\right)} + 2$
iii. $y = \sqrt{3}x^3 - 2x^2 - 0.5x + 1$	iv. $y = x^4 - x^2 + x$

31. Alex correctly determines that $(x-2)$ is a factor of $P(x) = 2x^4 - 13x^3 + 25x^2 - 8x - 12$.

- Determine the factors and write the equation $P(x)$ in its factored form.
- State the roots of the equation
- Explain how a graph of the function can be used to verify the roots.

$(x+0.5)(x-2)^2(x-3)$



32. Describe, using the words the series of transformations used to transform the graph of $f(x) = 2(x+1)^3$ to the graph of $g(x) = -(x-4)^3$.

reflected over the y-axis
compressed vertically by $\frac{1}{2}$
translated right by 3.

33. Determine the zeros of the function $f(x) = 2x^3 - x^2 - 18x + 9$ by factoring. Show your work.

$2(2)^3 - (3)^2 - 18(3) + 9$
$P(3) = 0$
$x = 3$ is a root
$x-3$

$2x^2 + 5x - 3$

x	$2x^3$	$+5x^2$	$-3x$
-3	$-6x^2$	$-15x$	$+9$

$2x - 1$

x	$2x^2$	$-1x$
3	$6x$	-3

$(x-3)(2x-1)(x+3)$

$x = -3$
$x = 3$
$x = \frac{1}{2}$

34. Given the function $P(x) = x^3 - 2x^2 - 5x + 6$

a. List all possible integral factors of $P(x)$. Explain how you determined this.
b. Algebraically determine the polynomial $P(x)$in completely factored form.
c. Determine the roots of $x^3 - 2x^2 - 5x + 6$.

1, -1
2, -2
3, -3
6, -6

$(3)^3 - 2(3)^2 - 5(3) + 6 = 0$

$x = 3$

$x - 3 = 0$

	x^2	$+1x$	-2
x	x^3	$+1x^2$	$-2x$
-3	$-3x^2$	$-3x$	$+6$

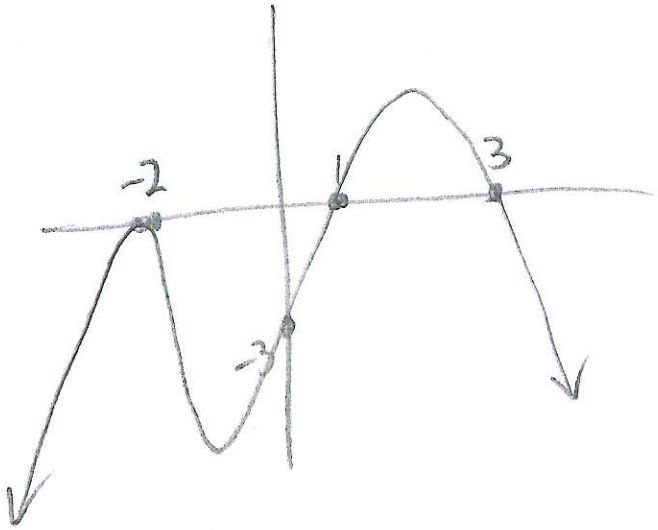
	x	-1
x	x^2	$-1x$
2	$+2x$	-2

$(x-3)(x+2)(x-1)$

$x = 3$
$x = -2$
$x = 1$

35. Sketch a graph of a polynomial function with the following characteristics, using the grid below.

- Degree of four
- Zeroes: -2 with a multiplicity of two
  1 with a multiplicity of one
  3 with a multiplicity of one
- y-intercept of -3
- end behaviour on the right is negative



36. Given the function $P(x) = x^3 - 2x^2 - 5x + 6$

a. List all possible integral factors of $P(x)$. Explain how you determined this.
b. Algebraically determine the polynomial $P(x)$in completely factored form.
c. Determine the solutions of $P(x) = 0$. Show all of your work.