Multiplicity & Graph Behaviour

1. A Subtle Misconception

A student claims:

"If a root has multiplicity greater than 1, the graph touches the x-axis twice."

a) Is this statement correct?

b) If not, provide a specific counterexample (in factored form).

c) Rewrite the statement so it is mathematically accurate.

2. Same Intercepts, Different Stories

Two polynomials both have x-intercepts at: $x = -3$ and $x = 2$.

- Function A crosses the x-axis at both intercepts.
- Function B crosses at $x = -3$ but flattens and crosses at $x = 2$.

a) What must be different about their multiplicities?

b) Give one possible factored form for each function.

c) Could the two functions have the same degree? Explain.

3. Sharp vs Flat

Two graphs both cross the x-axis at $x = 1$.

- Graph A crosses sharply.
- Graph B crosses slowly and flattens as it passes through.

a) Which graph likely has the higher multiplicity?

b) What multiplicities are possible in each case?

c) Sketch both behaviours.

4. Built to Touch

A polynomial:

- Touches the x-axis at $x = -4$
- Touches the x-axis at $x = 1$
- Has both ends pointing downward

a) What must be true about the multiplicity of each root?

b) What is the smallest possible degree?

c) Write one possible factored form.

5. Same Degree, Different Shapes

Both functions are degree 4.

Function A: $y = 2(x - 1)^2(x + 3)(x - 4)$

Function B: $y = -3(x - 1)(x + 3)^2(x - 4)$

a) Which roots are shared?

b) At which x-values do they behave differently? Why?

c) Compare their end behaviour.

d) Which graph could have more turning points? Explain.

6. Reverse Engineering

A graph behaves as follows:

- Crosses at $x = -2$
- Touches at $x = 0$
- Flattens and crosses at $x = 3$
- The right end goes up

a) Which roots have even multiplicity?

b) Which roots have odd multiplicity greater than 1?

c) What is the minimum possible degree?

d) Is the leading coefficient positive or negative? Explain.

7. Possible or Impossible?

For each description, state whether it is **possible or impossible**, and justify.

A) Degree 4 polynomial that touches three times

B) Degree 5 polynomial that crosses exactly twice

C) Degree 6 polynomial with one touching point and one crossing point only

D) Degree 7 polynomial that never crosses the x-axis

8. Limited Information

You are told:

- The polynomial has degree 7.
- It has exactly two distinct real roots.
- It crosses the x-axis exactly once.

a) What must the multiplicities be?

b) Could both ends point downward? Why or why not?

c) What can you conclude about the leading coefficient?

9. Degree Detective

A polynomial:

- Has exactly three distinct real roots
- Touches the x-axis at exactly one of them
- Crosses at the other two
- Has similar end behaviour
- Has no complex roots

a) Determine the multiplicity of each root.

b) What is the exact degree of the polynomial?

c) Explain why no other degree is possible.

10. Build Under Constraints

Construct a polynomial that:

- Crosses at $x = -1$
- Crosses at $x = 2$
- Touches at $x = 5$
- Has both ends up
- Contains a factor of the form $(2x - 3)$

a) Write one possible factored form.

b) State its degree.

c) Explain how you verified the end behaviour before expanding.