

Factoring & Sketching Polynomial Functions

(All functions are given in general form)

For each polynomial:

- Factor completely
- Determine the roots
- Determine the y-intercept
- Label the roots and y-intercept
- Sketch the function

1. $P(x) = x^3 + 5x^2 + 4x$

2. $P(x) = x^3 + x^2 - 6x$

3. $P(x) = x^3 + 4x^2 - 5x$

4. $P(x) = x^3 - 2x^2 - 8x$

5. $P(x) = 2x^3 + 6x^2 + 4x$

6. $P(x) = 3x^3 + 3x^2 - 6x$

7. $P(x) = -4x^3 + 14x^2 - 6x$

8. $P(x) = 6x^3 - 21x^2 - 6x$

9. $P(x) = x^3 - 4x^2 + 4x$

10. $P(x) = -2x^3 - 4x^2 - 2x$

11. $P(x) = 12x^3 - 12x^2 + 3x$

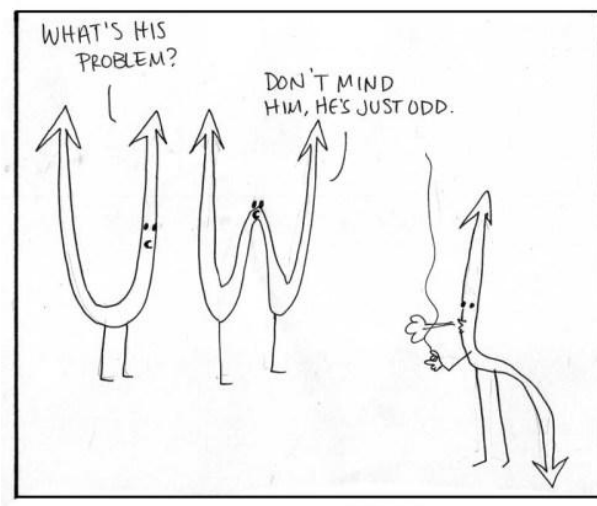
12. $P(x) = -6x^3 + 15x^2 - 12x$

13. $P(x) = x^4 - x^3 - 2x^2$

14. $P(x) = -4x^4 + 10x^3 + 2x^2$

15. $P(x) = 3x^4 - 12x^3 + 12x^2$

16. $P(x) = -12x^4 + 12x^3 - 3x^2$



Multiplicity of Polynomial Functions

1. Two functions have the same x-intercepts at $x = -2$ and $x = 1$.
 - Function A crosses the x-axis at both intercepts.
 - Function B crosses at $x = -2$ but only touches at $x = 1$.
 - a) What must be different about the factored form of these two functions?
 - b) Give one possible factored form for each function.
 - c) Which function has a higher degree? Explain.
2. A polynomial touches the x-axis at two different x-values and crosses nowhere.
 - a. What can you say about the multiplicity of each root?
 - b. Is this polynomial more likely to be even or odd degree? Why?
 - c. Sketch one possible graph that satisfies these conditions.
3. You are given the information below (no equation shown):
 - The graph crosses the x-axis at $x = -3$.
 - It flattens and crosses at $x = 1$.
 - It touches and turns at $x = 4$.
 - a. Which root has the highest multiplicity?
 - b. Which multiplicities are odd? Which are even?
 - c. What is the minimum possible degree of this polynomial?
4. A student claims: "If a graph touches the x-axis, the factor must be squared."
 - a) Is this statement always true?
 - b) If not, give a counterexample.
 - c) Rewrite the statement so it is mathematically correct.
5. Both of the following polynomials are **degree 4** and have **positive leading coefficients**.
 - Function A crosses the x-axis four times.
 - Function B crosses twice and touches once.
 - a) How is this possible if both have the same degree?
 - b) What must be true about the multiplicities of their roots?
 - c) Which graph is more likely to have more turning points? Explain.
6. For each statement, decide **true or false** and justify your answer.
 - a) A root with multiplicity 3 always changes direction.
 - b) A polynomial with only even multiplicities can cross the x-axis.
 - c) The graph of an odd-degree polynomial must cross the x-axis at least once.
 - d) Two different polynomials can have identical x-intercepts but different shapes.
7. Which of the following descriptions is **impossible**? Explain.
 - A) Degree 3 polynomial with two touching points
 - B) Degree 4 polynomial that crosses the x-axis exactly once
 - C) Degree 5 polynomial that touches the x-axis twice
 - D) Degree 6 polynomial with three crossing points