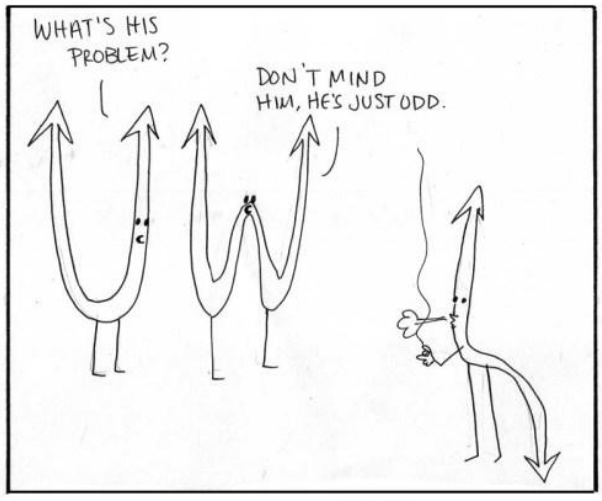
Factoring & Sketching Polynomial Functions

(All functions are given in general form)

For each polynomial:

- Factor completely
- Determine the roots
- Determine the y-intercept
- Label the roots and y-intercept
- Sketch the function

1. $P(x) = x^3 + 5x^2 + 4x$
2. $P(x) = x^3 + x^2 - 6x$
3. $P(x) = x^3 + 4x^2 - 5x$
4. $P(x) = x^3 - 2x^2 - 8x$
5. $P(x) = 2x^3 + 6x^2 + 4x$
6. $P(x) = 3x^3 + 3x^2 - 6x$
7. $P(x) = -4x^3 + 14x^2 - 6x$
8. $P(x) = 6x^3 - 21x^2 - 6x$
9. $P(x) = x^3 - 4x^2 + 4x$
10. $P(x) = -2x^3 - 4x^2 - 2x$
11. $P(x) = 12x^3 - 12x^2 + 3x$
12. $P(x) = -6x^3 - 27x^2 - 12x$
13. $P(x) = x^4 - x^3 - 2x^2$
14. $P(x) = -4x^4 + 10x^3 + 6x^2$
15. $P(x) = 3x^4 - 12x^3 + 12x^2$
16. $P(x) = -12x^4 + 12x^3 - 3x^2$



Multiplicity of Polynomial Functions

1. Two functions have the same x-intercepts at $x = -2$ and $x = 1$.
   - Function A crosses the x-axis at both intercepts.
   - Function B crosses at $x = -2$ but only touches at $x = 1$.

   a) What must be different about the factored form of these two functions?
   b) Give one possible factored form for each function.
   c) Which function has a higher degree? Explain.

2. A polynomial touches the x-axis at two different x-values and crosses nowhere.
   a. What can you say about the multiplicity of each root?
   b. Is this polynomial more likely to be even or odd degree? Why?
   c. Sketch one possible graph that satisfies these conditions.

3. You are given the information below (no equation shown):
   - The graph crosses the x-axis at $x = -3$.
   - It flattens and crosses at $x = 1$.
   - It touches and turns at $x = 4$.

   a. Which root has the highest multiplicity?
   b. Which multiplicities are odd? Which are even?
   c. What is the minimum possible degree of this polynomial?

4. A student claims: "If a graph touches the x-axis, the factor must be squared."
   a) Is this statement always true?
   b) If not, give a counterexample.
   c) Rewrite the statement so it is mathematically correct.

5. Both of the following polynomials are **degree 4** and have **positive leading coefficients**.
   - Function A crosses the x-axis four times.
   - Function B crosses twice and touches once.

   a) How is this possible if both have the same degree?
   b) What must be true about the multiplicities of their roots?
   c) Which graph is more likely to have more turning points? Explain.

6. For each statement, decide **true or false** and justify your answer.
   a) A root with multiplicity 3 always changes direction.
   b) A polynomial with only even multiplicities can cross the x-axis.
   c) The graph of an odd-degree polynomial must cross the x-axis at least once.
   d) Two different polynomials can have identical x-intercepts but different shapes.

7. Which of the following descriptions is **impossible**? Explain.
   A) Degree 3 polynomial with two touching points
   B) Degree 4 polynomial that crosses the x-axis exactly once
   C) Degree 5 polynomial that touches the x-axis twice
   D) Degree 6 polynomial with three crossing points