

# Formula Sheet – Pre-Calculus 12

## Linear Functions

### Calculating Slope

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

### y-intercept Form

$$y = mx + b$$

#### Sketching Method:

- Identify the slope  $m$  and the  $y$ -intercept  $b$ .
- Plot the  $y$ -intercept at  $(0, b)$ .
- Use the slope  $m = \frac{\text{rise}}{\text{run}}$  to plot a second point.
- Draw the line through the points.

### Slope-Point Form

$$y - y_1 = m(x - x_1)$$

#### Sketching Method:

- Identify the point  $(x_1, y_1)$  and the slope  $m$ .
- Plot the point on the graph.
- Use the slope  $m = \frac{\text{rise}}{\text{run}}$  to plot a second point.
- Draw the line through the points.

### General Form

$$Ax + By = C$$

#### Sketching Method:

- Find the  $y$ -intercept: set  $x = 0$  and calculate  $y$ .
- Find the  $x$ -intercept: set  $y = 0$  and solve for  $x$ .
- Plot both intercepts.
- Draw the line connecting them.

## Quadratic Functions

### Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Discriminant: } b^2 - 4ac$$

### General Form

$$y = ax^2 + bx + c$$

#### Sketching Method

- Find the **axis of symmetry** using  $x = -\frac{b}{2a}$
- Find the **vertex** with  $f$  (axis of symmetry).
- Find the **y-intercept** with  $f(0)$ .

#### Sketching Method 2

- Identify  $a$ ,  $b$ , and  $c$ .
- Identify the **x-intercepts**:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Find the **axis of symmetry** by averaging the x-intercepts.
- Find the **vertex** with  $f$  (axis of symmetry).

### Standard (Vertex) Form

$$y = a(x - p)^2 + q$$

#### Sketching Method 1

- Read the vertex  $(h, k)$ .
- Draw the axis of symmetry, vertically through the vertex.
- Find the **y-intercept** with  $f(0)$ .

### Factored Form

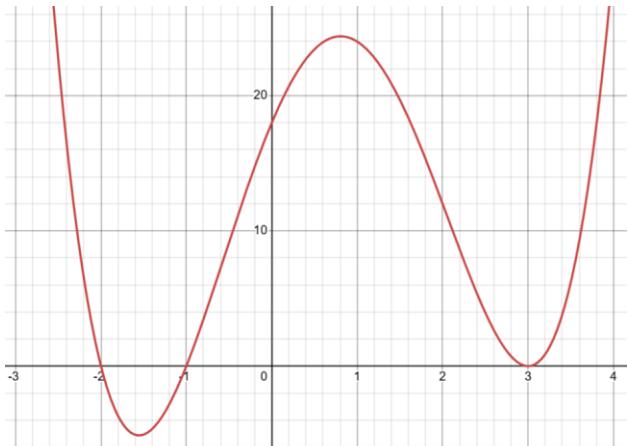
$$y = a(x - r_1)(x - r_2)$$

#### Sketching Method

- Identify the **x-intercepts** by solving for  $x$  in each of the binomials.
- Find the **axis of symmetry** by averaging the x-intercepts.
- Find the **vertex** with  $f$  (axis of symmetry).

## Polynomial Functions

$$y = a(x - 3)^2(x + 2)(x + 1)$$



y-intercept  $\rightarrow$  crosses the y-axis at  $x = 0$

x-intercept  $\rightarrow$  roots  $\rightarrow$  crosses at  $y = 0$

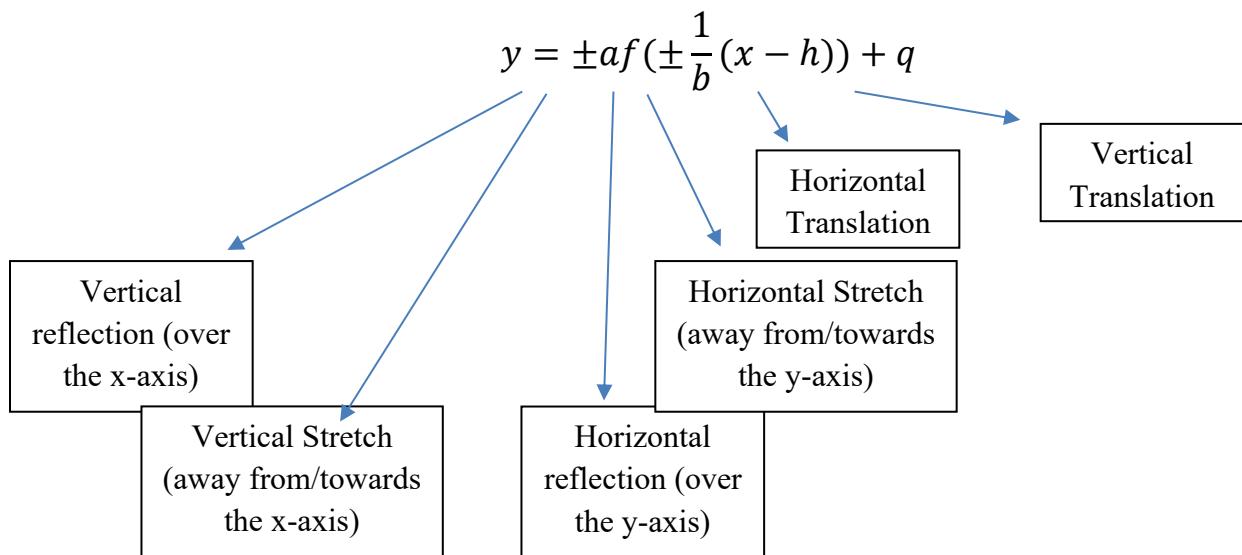
factors  $\rightarrow (x - \text{root}) = 0$

$a \rightarrow$  Vertical Stretch

multiplicity  $\rightarrow$  repeated factors

local max / local min  $\rightarrow$  peaks & troughs

## Function Transformations



# Exponents & Logarithms Formula Sheet

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## Laws of Exponents

$$a^x a^y = a^{x+y}$$

$$(a^m)^n = a^{mn}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^y b^y = (ab)^y$$

## Radical Properties

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$a\sqrt{b} = \sqrt{a^2b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

## Laws of Logarithms

$$\log_a b = \frac{\log b}{\log a}$$

$$\log(ab) = \log a + \log b$$

$$\log a^m = m \log a$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

## Natural Logarithm

$$e^{\ln x} = x$$

$$\ln e^x = x$$

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## Arithmetic Sequences

$$t_n = t_1 + (n-1)d$$

$$S_n = \frac{n(t_1 + t_n)}{2}$$

## Geometric Sequences

$$t_n = ar^{n-1}$$

$$r = \frac{t_2}{t_1}$$

## Applications of Geometric Sequences

$$\text{Compound Interest} \rightarrow A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$\text{Population} \rightarrow P_f = P_i r^{\text{periods}}$$

$$\text{Half Lives} \rightarrow m_f = m_i 0.5^{\frac{t}{\text{half-life}}}$$

$$\text{Investment} \rightarrow A = P(r)^{\text{periods}}$$

$$\text{Earthquake Intensity} \rightarrow I = 10^{R_F - R_o}$$

$$\text{Sound Intensity} \rightarrow I = 10^{\frac{D_f - D_o}{10}}$$

$$\text{pH} \rightarrow [H^+] = (10)^{-pH}$$

## Geometric Series

$$S_n = a \frac{(1 - r^n)}{1 - r}$$

$$S_{n+1} - S_n = t_{n+1}$$

## Infinite Geometric Series

$$S_\infty = a \left( \frac{1}{1 - r} \right)$$

## Ordinary Annuity

$$S_n = A \frac{(1 + r)^n - 1}{r}$$

## Annuity Due

$$S_n = A(1 + r) \frac{(1 + r)^n - 1}{r}$$

## Continuous Growth Rate

$$e^r = \left(1 + \frac{r}{n}\right)^n$$

$$A = P(e^r)^t$$

$$S_n = A e^r \left( \frac{e^{rn} - 1}{e^r - 1} \right)$$

# Trigonometry Formula Sheet

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## Angles in Standard Position

Vertex at origin & Initial arm on positive  $x$ -axis

## Reference Angle

Acute angle between terminal arm and  $x$ -axis

## Coterminal Angles

$$\theta = \theta + 360^\circ k$$

## Angle Conversion

$$\frac{\text{Radians}}{\text{Degrees}} = \frac{2\pi}{360}$$

## Function Transformations

$$y = A \sin(B(x - C)) + D \quad \text{and} \quad y = A \cos(B(x - C)) + D \quad \text{where} \quad B = \frac{2\pi}{p}$$

## Right Triangle Ratios (SOH-CAH-TOA)

$$\sin \theta = \frac{o}{h} \quad \cos \theta = \frac{a}{h} \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

## Cosine Law

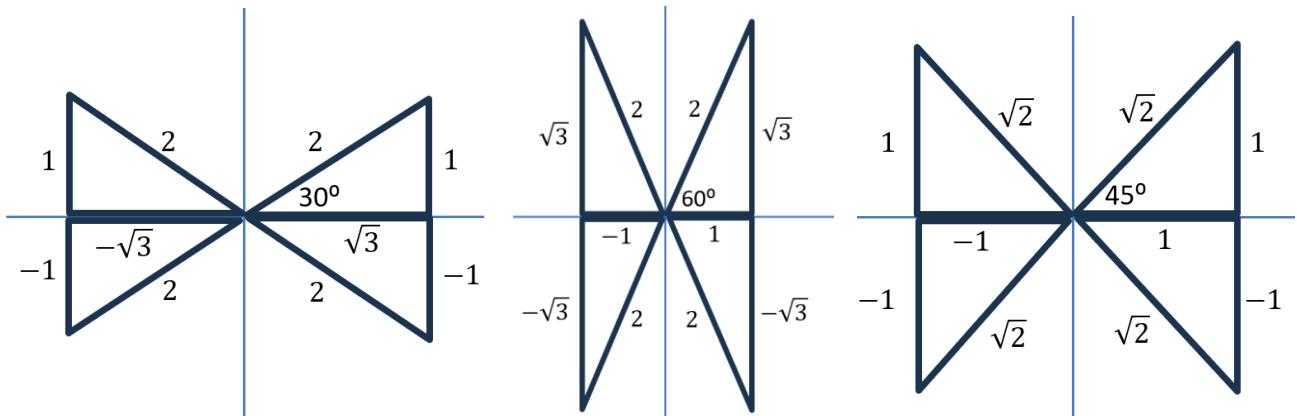
$$c^2 = a^2 + b^2 - 2ab \cos C$$

## Sine Law

$$a \sin B = b \sin A \quad \frac{a}{\sin A} = \frac{b}{\sin B} \quad \frac{\sin A}{a} = \frac{\sin B}{b}$$

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## Special Triangles & Bowties



## Trigonometric Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$1 + \cos \theta$     OR     $1 - \cos \theta$      $\rightarrow$     Multiply by Conjugate

$1 + \sin \theta$     OR     $1 - \sin \theta$      $\rightarrow$     Multiply by Conjugate

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## Sum & Difference Angle Identities

### Sine:

$$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

### Cosine:

$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

### Tangent:

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

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## Double Angle Identities

### Sine:

$$\sin(2A) = 2\sin(A)\cos(A)$$

### Tangent:

$$\tan(2A) = \frac{2\tan(A)}{1 - \tan^2(A)}$$

### Cosine:

$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

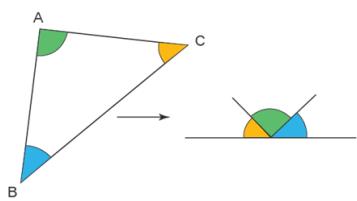
$$\cos(2A) = 2\cos^2(A) - 1$$

$$\cos(2A) = 1 - 2\sin^2(A)$$

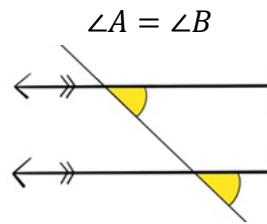
# Common Angle Relationships

## Triangle Angle Sum

$$\angle A + \angle B + \angle C = 180^\circ$$

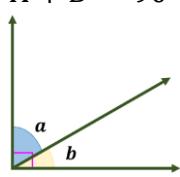


## Corresponding Angles



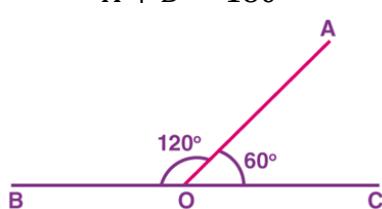
## Complementary Angles

$$A + B = 90^\circ$$



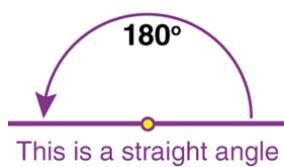
## Supplementary Angles

$$A + B = 180^\circ$$

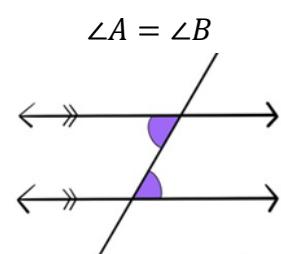


## Straight Angles

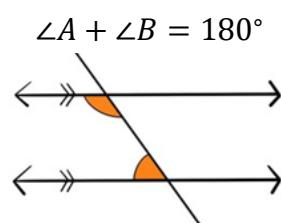
$$\text{Straight angle} = 180^\circ$$



## Alternate Interior Angles

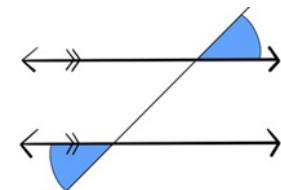


## Co-Interior Angles



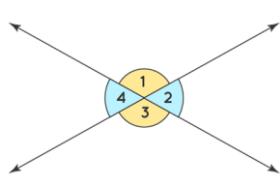
## Alternate Exterior Angles

$$\angle A = \angle B$$



## Vertically Opposite Angles

$$\angle A = \angle B$$



## Co-Exterior Angles

$$\angle A + \angle B = 180^\circ$$

