# Polynomial Function Problems

1. Which of the following binomials is a factor of the polynomial $P(x) = 2x^3 + 5x^2 - 28x - 15$

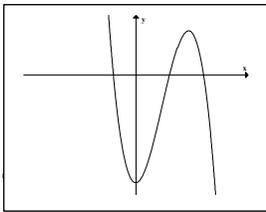    A. $2x - 3$                 B. $x + 3$                 C. $2x - 1$                 D. $x + 5$

2. In a polynomial $P(x)$, $P\left(-\frac{2}{5}\right) = 0$. One of the factors of $P(x)$ is

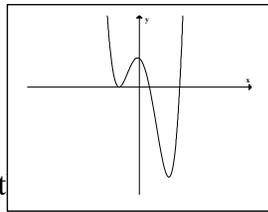    A. $(5x - 2)$            B. $(2x - 5)$            C. $(5x + 2)$            D. $(2x + 5)$

3. If $x + 2$ is a factor of P(x), then the value of P(-2) is:

    A. 0                    B. $-\frac{1}{2}$              C. 2                 D. $-2$

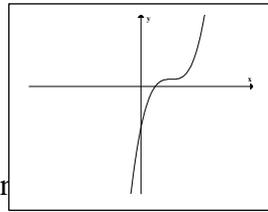4. Sam is presented with the following four partial graphs of polynomial functions, and is asked to select the graphs with an odd degree.
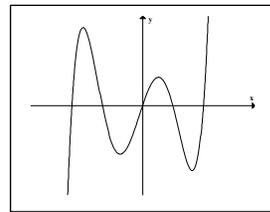


    A. 1                      B. 2                   C. 3                   D. 4

5. The remainder when $P(y) = 2y^4 - 3y^2 + 1$ is divided by $(y - 3)$ is _____.

6. Sketch a graph of a polynomial function with the following characteristics, using the grid below.

    a) Degree of four
    b) Zeroes: -2 with a multiplicity of two
                1 with a multiplicity of one
                3 with a multiplicity of one
    c) $y$-intercept of -3
    d) end behaviour on the right is negative

7. When $2x^5 - 3x^3 - 8x^2 - 8$ is divided by $x - 2$, the sum of the coefficients of the terms in the quotient is

    A. $-59$
    B. $-23$
    C. $-19$
    D. $17$

8. If $P(x) = x^3 - 7x - 3$ is divided by $(x + 1)$, then $P(x)$ may be written as

    A. $(x^2 - 8)(x + 1) + 5$
    B. $(x^2 - x - 6)(x + 1) + 3$
    C. $(x^3 - 7x - 3)(x + 1) + 3$
    D. $(x^3 - 7x - 3)(x + 1) + 5$

9. If $-5x$ is a factor of the polynomial $P(x)$, then $P(0)$ is

    A. $-5$
    B. $0$
    C. $\dfrac{1}{5}$
    D. $5$

10. If $P(x)$ is a cubic polynomial function with $P(1) = P(3) = P(-4) = 0$ and $P(0) = 36$, then $P(x)$ is

    A. $-3(x - 1)(x - 3)(x + 4)$
    B. $-2(x + 1)(x + 3)(x - 4)$
    C. $2(x + 1)(x + 3)(x - 4)$
    D. $3(x - 1)(x - 3)(x + 4)$

11. If the polynomial function $P(x) = 3x^3 - 9x^2 + kx - 12$ is divisible by $x - 3$, then it is also divisible by

    A. $3x - 4$
    B. $3x + 4$
    C. $3x^2 + 4$
    D. $3x^2 - 4$

12. The area of a rectangle is $(x^3 + 5x^2 - 4x - 20)$ cm$^2$ and the width is $(x + 2)$ cm. If the length of the rectangle is 8 cm, then the width in centimetres is

    A. 3 cm
    B. 5 cm
    C. 6 cm
    D. 8 cm

13. If $(x - 3)$ is a factor of the polynomial function $P$, where $P(x) = x^3 - 2x^2 + kx + 6$, then the value of $k$ is

    A. $-13$
    B. $-5$
    C. $5$
    D. $13$

14. An integral polynomial function is defined by $P(x) = x^3 - ax^2 - 2x + 2a$, $a \neq 0$. Which of the following is a factor of $P(x)$ for any value of $a$?

    A. $x - a$
    B. $x + \sqrt{a}$
    C. $x - \sqrt{a}$
    D. $x + a$

15. The polynomial function $P(x) = 4x^3 - 18x^2 - x - 45$ is divided by $D(x) = x - 5$. The quotient is of the form $Q(x) = ax^2 + bx + c$. Determine the values of $a$, $b$, and $c$, where $a$, $b$, and $c$.

16. The polynomial $f(x) = x^3 - x^2 - 40x + n$ is divisible by $x - 3$. The value of n, correct to the nearest whole number, is _____.

17. A factor of $4x^3 - 8x^2 + 5x - 1$ is $x - 1$. The other first degree factors of this polynomial are

    a. $(2x - 1) and (2x + 1)$
    b. $(2x - 1) and (2x - 1)$
    c. $(4x - 1) and (x + 1)$
    d. $(x - 1) and (x + 1)$

18. A step in the process of factoring the polynomial $P(x) = x^5 - x^3 + 2x - 2$ is

    a. $(x - 1)(x^4 - 2x^2)$
    b. $(x - 1)(x^4 + x^3 + 2)$
    c. $(x + 1)(x^4 - x^3 + 2)$
    d. $(x + 1)(x^4 - x^3 - 2x^2 - 2x - 4)$

19. For the polynomial $P(x) = 3x^3 - 2x^2 - 7x - 2$, two of the zeros are 1 and 2. The third zero for this polynomial is

    a. $\frac{5}{3}$
    b. $\frac{2}{3}$
    c. $\frac{1}{3}$
    d. $-\frac{1}{3}$

20. A student graphed the equation of a polynomial $y = P(x)$ to help determine the solution to the equation $P(x) = 0$. Which of the following provides the solution to $P(x) = 0$.

    a. The x intercepts of the graph of $y = P(x)$
    b. The y intercepts of the graph of $y = P(x)$
    c. The point(s) of intersection of $x = 0$ and the graph of $y = P(x)$
    d. The point(s) of intersection of $y = 0$ and the graph of $P(0)$

21. Which of the following functions is a polynomial function?

    a. $f(x) = 2^x + 6x^2 + 5$
    b. $f(x) = x^2 + 6x + \sqrt{5}$
    c. $f(x) = x^{\frac{2}{3}} + 5x^2 - 7$
    d. $f(x) = \frac{x^3}{3} + \sqrt{5}x - \frac{1}{x}$

22. Sketch a graph of a polynomial function with the following characteristics, using the grid below.

    a) Degree of four
    b) Zeroes: -2 with a multiplicity of two
               1 with a multiplicity of one
               3 with a multiplicity of one
    c) $y$-intercept of -3
    d) end behaviour on the right is negative

23. The polynomial $f(x) = x^3 - 7x^2 + 2x + n$ is divisible by $x + 2$. The value of n, correct to the nearest whole number, is _____.

24. A factor of $4x^3 - x^2 - 4x + 1$ is $x - 1$. The other first degree factors of this polynomial are

    a. $(2x - 1) and (2x + 1)$
    b. $(2x - 1) and (2x - 1)$
    c. $(4x - 1) and (x + 1)$
    d. $(x - 1) and (x + 1)$

25. A student graphed the equation of a polynomial $y = P(x)$ to help determine the solution to the equation $P(x) = 0$. Which of the following provides the solution to $P(x) = 0$.

    a. The point(s) of intersection of $x = 0$ and the graph of $y = P(x)$
    b. The point(s) of intersection of $y = 0$ and the graph of $P(0)$
    c. The x intercepts of the graph of $y = P(x)$
    d. The y intercepts of the graph of $y = P(x)$

26. The factors of the polynomial $P(x) = 6x^3 + x^2 - 10x + 3$ are $(2x+3), (3x-1)$, and $(x-1)$. If the polynomial $P(x)$ is multiplied by -2, then the result is the polynomial $-2P(x)$. The zeroes of the new function are

   a) $\dfrac{-3}{2}, \dfrac{1}{3}, 1$

   b) $\dfrac{3}{2}, -\dfrac{1}{3}, -1$

   c) $-3, \dfrac{2}{3}, 2$

   d) $3, -\dfrac{2}{3}, -2$

27. Which statement is true?
   a. Some odd degree polynomial functions have no x-intercepts
   b. Even-degree polynomial functions always have an even number of x-intercepts
   c. All odd-degree polynomial functions have at least one x-intercept
   d. All even degree polynomial functions have at least one x-intercept

28. A student graphed the equation of a polynomial function, $y = P(x)$, to help determine the solution to the equation $P(x) = 0$. Which of the following provides the solution to $P(x) = 0$?

   A. The x-intercepts of the graph of $y = P(x)$.
   B. The y-intercept of the graph of $y = P(x)$.
   C. The point(s) of intersection of $x = 0$ and the graph of $y = P(x)$.
   D. The point(s) of intersection of $y = 0$ and $P(0)$.

29. The equation of a polynomial function is $P(x) = kx(x+2)(x-6)$, $k \neq 0$. If a new function results from doubling the value of $k$ only, then the zeros of the new function

   A. remain unchanged
   B. are doubled
   C. are 0, –1, and 3
   D. are 0, –4, and 12

30. Circle the polynomial functions below. Explain why the other(s) are not polynomial functions.

   i. $y = \dfrac{3}{4}x^{-5} - 1$         ii. $y = 3x^4 - x^{\frac{1}{2}} + 2$

   iii. $y = \sqrt{3}x^3 - 2x^2 - 0.5x + 1$      iv. $y = x^4 - x^2 + x$

31. Alex correctly determines that $(x - 2)$ is a factor of $P(x) = 2x^4 - 13x^3 + 25x^2 - 8x - 12$.

- Determine the factors and write the equation $P(x)$ in its factored form.
- State the roots of the equation
- Explain how a graph of the function can be used to verify the roots.

32. Describe, using the words the series of transformations used to transform the graph of $f(x) = 2(x + 1)^3$ to the graph of $g(x) = -(x - 4)^3$.

33. Determine the zeros of the function $f(x) = 2x^3 - x^2 - 18x + 9$ by factoring. Show your work.

34. Given the function $P(x) = x^3 - 2x^2 - 5x + 6$

a. List all possible integral factors of $P(x)$. Explain how you determined this.
b. Algebraically determine the polynomial $P(x)$ in completely factored form.
c. Determine the roots of $x^3 - 2x^2 - 5x + 6$.

35. Sketch a graph of a polynomial function with the following characteristics, using the grid below.
- Degree of four
- Zeroes: -2 with a multiplicity of two
    1 with a multiplicity of one
    3 with a multiplicity of one
- $y$-intercept of -3
- end behaviour on the right is negative

36. Given the function $P(x) = x^3 - 2x^2 - 5x + 6$
a. List all possible integral factors of $P(x)$. Explain how you determined this.
b. Algebraically determine the polynomial $P(x)$ in completely factored form.
c. Determine the solutions of $P(x) = 0$. Show all of your work.